2012 Marywood Mathematics Contest

Level I

Sponsored by

SEMI-GROUP

The Student Mathematics Club of

Marywood University

March 24, 2012

Directions:

1. This exam consists of 40 questions on 5 pages. Please check to make sure that you have all the pages.

2. No calculator or any other computing device is allowed on this exam.

3. Allot your time accordingly. This is a 60-minute test. Do not spend too much time on any one problem. If a question seems to be too difficult, make your best possible guess. Your score will be the number of correct responses.

4. On the scantron form provided for you, darken in the space corresponding to the correct answer. Please mark all answers carefully and erase completely when changing an answer. Mark **only one answer** for each question. Only those answers on the answer sheet will be counted.

5. There is a sheet of blank paper on the last page which you can tear off and use as scratch paper. You may also use the back of the pages.

6. NOTE: **In order to ensure uniformity, proctors are NOT allowed to answer any questions pertaining to specific problem content.**

Please do NOT open the test until you are told to do so.

1. Write fifteen-and-a-half billion in scientific notation $a \times 10^n$. What is the value of n?

   A. 8   **B.** 9   **C.** 10   **D.** 11   **E.** 12

2. The positive difference between $1/2$ and its reciprocal is

   A. $3/2$ **B.** $1/2$ **C.** $5/2$ **D.** 1   **E.** None of these.

3. $1^2 + 3^4 =$

   A. $1^4 + 3^2$ **B.** $1^3 + 4^2$ **C.** $1^2 + 4^3$ **D.** $1^3 + 2^4$ **E.** None of these.

4. The product of 2012 distinct integers is an even number, **at most** how many of those 2012 integers can be odd?

   A. 0   **B.** 1   **C.** 2011   **D.** 2012   **E.** None of these.

5. $(a + b + c)^2 =$

   A. $a^2 + b^2 + c^2$ **B.** $a^2 + b^2 + c^2 + 2ab + 2ac$ **C.** $a^2 + b^2 + c^2 + 2ab + 2bc$

   D. $a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$ **E.** None of these.

6. $2^{2^{2^2}} =$

   A. 16   **B.** 64   **C.** 128   **D.** 256   **E.** None of these.

7. The area of a circle with diameter 6 is

   A. 6π **B.** 12π **C.** 18π **D.** 36π **E.** None of these.

8. $11 \times 22 \times 33 \times 44 \times 55 \times 66 \times 77 \times 88 \times 99 = 9! \times$

   A. 11^9 **B.** 11×9 **C.** 11^{10} **D.** 11×10 **E.** None of these.

9. If $3x + 24 = 7x + 2012$, then $x =$

   A. 497   **B.** -497 **C.** 506   **D.** -506 **E.** None of these.

10. In $\triangle ABC$, $\angle A$ measures $40°$, and $\angle B$ measures 6 times as $\angle C$. What is $m\angle B - m\angle C$?

    A. $20°$ **B.** $100°$ **C.** $120°$ **D.** $140°$ **E.** None of these.

11. If $\sqrt{36} + \sqrt{x} = \sqrt{100}$, then $x =$

    A. 2   **B.** 4   **C.** 8   **D.** 64   **E.** None of these.

12. $\log_3 81 =$

    A. 4   **B.** 27   **C.** 9   **D.** 2   **E.** None of these.

13. The slope of the line through the points $(3, 24)$ and $(20, 12)$ is

    A. $\dfrac{12}{17}$ **B.** $\dfrac{17}{12}$ **C.** $-\dfrac{12}{17}$ **D.** $-\dfrac{17}{12}$ **E.** None of these.

14. $\sqrt{\sqrt{16 \times 16 \times 16 \times 16}} =$

    A. 2   **B.** 4   **C.** 8   **D.** 16   **E.** None of these.

15. $5^{7x} = 25^{3x+100}$, then $x =$

    A. 25   **B.** 50   **C.** 100   **D.** 200   **E.** None of these.

16. If two different students are randomly selected from a class of 12 boys and 13 girls, what is the probability that both students are boys?

    A. 22%   **B.** 23%   **C.** 24%   **D.** 26%   **E.** None of these.

17. How many **odd** numbers between 1 and 1,000,000 (inclusive) are perfect **cubes**?

    A. 49   **B.** 50   **C.** 51   **D.** 99   **E.** 100

18. If a rectangle has a perimeter of 60 inches, and its length is 5 times the width, then the area of the rectangle is

    A. 125 in^2 **B.** 225 in^2 **C.** 250 in^2 **D.** 500 in^2 **E.** None of these.

19. The radius of the circle $x^2 + y^2 + 8x = 0$ is

    A. 2   **B.** 4   **C.** 8   **D.** 16   **E.** None of these.

20. The two lines $3x - y = 11$ and $x + 2y = -1$ intersect at the point (x_0, y_0). What is the product of x_0 and y_0?

    A. 6   **B.** -6   **C.** 1   **D.** -1   **E.** None of these.

21. $2^{2012} + 2^{2012} + 3^{2012} + 3^{2012} + 3^{2012} =$

    A. $4^{2012} + 9^{2012}$ **B.** $2^{4024} + 3^{6036}$ **C.** $4^{2013} + 9^{2013}$ **D.** $2^{2013} + 3^{2013}$ **E.** None of these.

22. The fraction $\dfrac{a}{b}$ is equal to $\dfrac{1}{8}$, and $a + b = 27$. What is the value of $a \times b$?

   **A.** 24   **B.** 8   **C.** 324   **D.** 72   **E.** None of these.

23. Which of the following is NOT the sum of three consecutive integers?

    A. 444   **B.** 12   **C.** 48   **D.** 76   **E.** None of these.

24. If a building has 6 entrances, in how many different ways can a person enter through one entrance and exit through a different entrance?

    A. 36   **B.** 30   **C.** 18   **D.** 15   **E.** None of these.

25. In the figure below, $AB = 5$, $CD = 9$, and the height of the trapezoid $AF = BE = 4$. What is the area of the shaded region in the trapezoid?
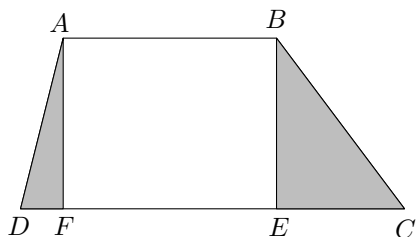
    A. 8

    B. 18

    C. 36

    D. 16

    E. None of these.

26. Alina took 10 quizzes and had an average of 94% for all 10. If her average on the first 6 quizzes was 92%, what was her average on the last 4 quizzes?

    A. 95%   **B.** 96%   **C.** 97%   **D.** 98%   **E.** None of these.

27. What is the degree measure of the angle formed by the hour hand and the minute hand of a clock at 9:30?

    A. 60°   **B.** 90°   **C.** 105°   **D.** 75°   **E.** None of these.

28. If a fair coin is tossed four times, what is the probability that no two consecutive tosses have the same results?

    A. $\dfrac{1}{16}$ **B.** $\dfrac{1}{8}$ **C.** $\dfrac{1}{4}$ **D.** $\dfrac{1}{2}$ **E.** None of these.

29. Notice that $324 = 2^2 \times 3^4$. How many positive divisors does 324 have?

    A. 8   **B.** 10   **C.** 12   **D.** 15   **E.** None of these.

30. The sum of the two roots of the equation $x^2 - 3x - 28 = 0$ is

    A. 3   **B.** -3 **C.** 11   **D.** -11 **E.** None of these.

31. A circle and a square have the same area, what is the ratio between the circumference of the circle and the perimeter of the square?

    A. $1:1$ **B.** $\sqrt{\pi}:2$ **C.** $\pi:2$ **D.** $1:\sqrt{\pi}$ **E.** $2:\sqrt{\pi}$

32. How many rectangles are there in the 2×4 grid below?
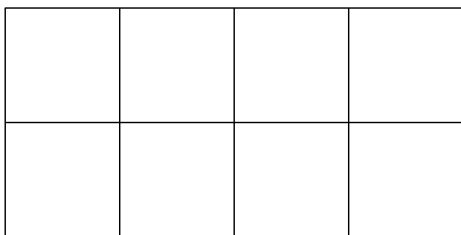
    A. 12
    B. 17
    C. 28
    D. 30
    E. None of these.

33. Two standard six-sided dice are rolled. What is the probability that the two numbers rolled will differ by exactly 2?

    A. $\dfrac{4}{11}$ **B.** $\dfrac{8}{11}$ **C.** $\dfrac{2}{9}$ **D.** $\dfrac{1}{9}$ **E.** None of these.

34. $10 \div 0.\overline{2} =$

    A. 5   **B.** 20   **C.** 45   **D.** 50   **E.** None of these.

35. What is the last digit of the number 7^{2012}?

    A. 1   **B.** 3   **C.** 7   **D.** 9   **E.** None of these.

36. During March Madness, the NCAA men's basketball tournament features 64 teams in a single elimination bracket. After the first round, the winner of each game moves on to the second round, where 32 teams compete, and the winner of each second round game moves on to the third round (sweet 16), and so on. Two friends are trying to predict the outcomes of all the games in the tournament. For each first round game predicted correctly, one gets 1 point. For each second round game predicted correctly, one gets 2 points. For each third round game predicted correctly, one gets 4 points, and so on (with the point value per correctly predicted game doubled at the successive rounds). If a person magically predicts ALL of the outcomes correctly, how many points would this person get?

A. 80 **B.** 160 **C.** 192 **D.** 224 **E.** None of these.

37. In the figure below, how many paths are there from A to Z, if the paths must trace the line segments downward and no retracing is allowed?
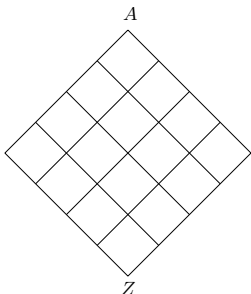
A. 20

B. 35

C. 55

D. 70

E. None of these.

38. A circular cone has the same height as a circular cylinder, but the base circumference of the cone is twice of the base circumference of the cylinder. What is the ratio of the volume of the cone to the volume of the cylinder?

A. $2:1$ **B.** $2:3$ **C.** $1:2$ **D.** $3:2$ **E.** None of these.

39. A positive integer will be called an *awesome-five* number if it does not contain the digit zero and all the digits sum up to 5. For example, 1121 is an *awesome-five* number, but 2012 and 324 are not. How many *awesome-five* numbers are there?

A. 14 **B.** 15 **C.** 16 **D.** 17 **E.** None of these.

40. What is the sum of the remainders when 100, 101, 102, ..., 998, 999, and 1000 are each divided by 9?

A. 3600 **B.** 3601 **C.** 4500 **D.** 4501 **E.** None of these.

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