

# 2013 Marywood Mathematics Contest

Level I

Sponsored by

**SEMI-GROUP**

The Student Mathematics Club of

Marywood University

March 16, 2013

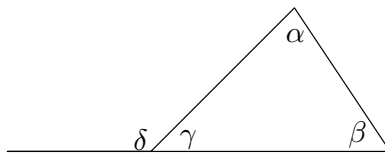
Directions:

1. This exam consists of 40 questions on 6 pages. Please check to make sure that you have all the pages.
2. No calculator or any other computing device is allowed on this exam.
3. Allot your time accordingly. This is a 60-minute test. Do not spend too much time on any one problem. If a question seems to be too difficult, make your best possible guess. Your score will be the number of correct responses.
4. On the scantron form provided for you, darken in the space corresponding to the correct answer. Please mark all answers carefully and erase completely when changing an answer. Mark **only one answer** for each question. Only those answers on the answer sheet will be counted.
5. There is a sheet of blank paper on the last page which you can tear off and use as scratch paper. You may also use the back of the pages.
6. NOTE: In order to ensure uniformity, proctors are NOT allowed to answer any questions pertaining to specific problem content.

Please do NOT open the test until you are told to do so.

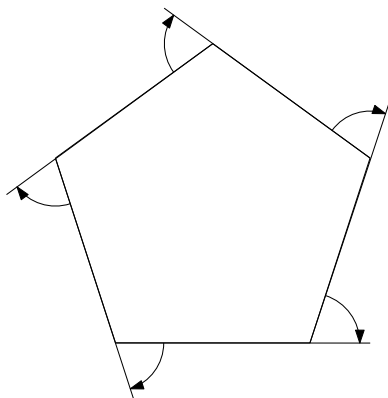
1. What is the average amount of time you have per problem to complete this exam?  
A. 30 seconds    B. 1 minute    C. 90 seconds    D. 2 minutes    E. 150 seconds
2. Solve the equation  $3x + 5 = 2(x + 7)$  for  $x$ .  
A. 1    B. 3    C. 5    D. 7    E. None of these.
3. If the radius of a circle is a rational number, its area is given by a(n) \_\_\_\_\_ number.  
A. rational    B. irrational    C. integral    D. whole    E. None of these.
4. The average of 4 and its reciprocal is  
A. 4.25    B. 2.125    C. 3.75    D. 0.25    E. None of these.
5. Order the numbers 3.1,  $\frac{22}{7}$ , and  $\pi$  from largest to smallest.  
A. 3.1,  $\pi$ ,  $\frac{22}{7}$     B.  $\frac{22}{7}$ , 3.1,  $\pi$     C.  $\pi$ , 3.1,  $\frac{22}{7}$     D.  $\pi$ ,  $\frac{22}{7}$ , 3.1    E. None of these.
6. In  $\triangle ABC$ ,  $m\angle B = 3 \cdot m\angle A$  and  $m\angle C = \frac{1}{6} \cdot m\angle B$ . Then  $m\angle A =$   
A.  $10^\circ$     B.  $20^\circ$     C.  $30^\circ$     D.  $40^\circ$     E. None of these.
7. If  $(x, y)$  is the intersection of the two lines  $3x - 2y = -2$  and  $-2x + 3y = 8$ , then  $x + y =$   
A. 6    B. 7    C. 8    D. 9    E. None of these.
8.  $3^{2^3} =$   
A. 6561    B. 729    C. 512    D. 216    E. None of these.
9. If  $\left[25^4 \cdot \frac{1}{27^2}\right]^{-1/2} = 5^a \cdot 3^b$ , then  $a + b =$   
A. -2    B. -1    C. 1    D. 2    E. None of these.
10. The distance between the points  $(2, 7)$  and  $(5, 3)$  is  
A. 3    B. 4    C. 5    D. 6    E. None of these.

11. The maximum value of  $y = x^2 - 2x - 15$  is
- A.  $-16$       B.  $1$       C.  $-3$       D.  $5$       E. None of these.
12. The coefficient of  $a^2b^2$  in the expansion of  $(a + b)^4$  is
- A.  $3$       B.  $4$       C.  $5$       D.  $6$       E. None of these.
13. The new binary operator  $\star$  is defined by  $a \star b = a \cdot b - a - b$ . What is  $(1 \star 2) \star 3$ ?
- A.  $-7$       B.  $-5$       C.  $-3$       D.  $-1$       E. None of these.
14. The sum of the roots of the quadratic  $2x^2 - 6x + 1$  is
- A.  $1$       B.  $2$       C.  $3$       D.  $4$       E. None of these.
15. Which of the following relationships must hold in the following figure?



- A.  $\alpha + \beta = \delta$     B.  $\alpha + \beta < \delta$     C.  $\alpha + \beta > \delta$     D.  $\delta = \beta$       E. None of these.
16. You are playing Dungeons and Dragons with your friend. Her orc will kill your troll unless you roll a higher number than she does. Since your friend's orc is more experienced than your measly troll, she can roll an eight sided die (d8) while you are only allowed to roll a six sided die (d6). Which of the following fractions **best represents** the chance that you will roll a higher number than your friend thus allowing your troll the opportunity to live and fight again?
- A.  $\frac{1}{5}$       B.  $\frac{1}{4}$       C.  $\frac{1}{3}$       D.  $\frac{1}{2}$       E.  $\frac{3}{4}$
17. Successive discounts of 10% and 20% are equivalent to a single discount of
- A. 30%      B. 15%      C. 72%      D. 28%      E. None of these.
18. If the radius of a circle is increased by 100%, the area of the circle will increase by
- A. 100%      B. 200%      C. 300%      D. 400%      E. None of these.

19. As the number of sides of a polygon increases from 3 to  $n$ , the sum of the exterior angles formed by extending each side in succession



- A. remains constant  
 B. increases  
 C. decreases  
 D. cannot be predicted  
 E. None of these.

20. A six place number is created by repeating any three digit number; for example, 256,256 or 678,678, etc. Any number of this form is always exactly divisible by

- A. 7 only      B. 11 only      C. 13 only      D. 101      E. 1001

21. The formula which expresses the relationship between  $x$  and  $y$  as shown in the accompanying table is

$x$	0	1	2	3	4
$y$	100	90	70	40	0

- A.  $y = 100 - 10x$       B.  $y = 100 - 5x^2$       C.  $y = 100 - 5x - 5x^2$   
 D.  $y = 20 - x - x^2$       E. None of these.

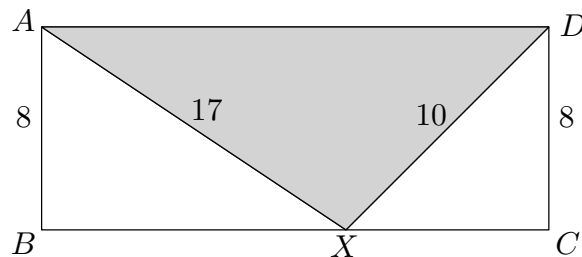
22. A polynomial of degree 6 has  $1 + 5i$  and  $2 + 3i$  as the only roots whose imaginary part is positive. How many real roots does the polynomial have?

- A. 4      B. 3      C. 2      D. 1      E. 0

23. Cool Clint drives 120 miles to take a math test at Marywood on a Saturday. Due to construction on I-81, his average speed on the way to Scranton is only 30 miles per hour. However, on the way home things have picked up and he is able to average 40 miles per hour. The average speed for his round trip is closest to

- A. 34 mph      B. 35 mph      C. 36 mph      D. 37 mph      E. 38 mph

24. What is the last digit of  $2013^{2013}$ ?
- A. 1            B. 3            C. 7            D. 9            E. None of these.
25. The number of circular pipes with an inside diameter of 1 inch which will carry the same amount of water as a pipe with an inside diameter of 6 inches is
- A.  $6\pi$             B. 6            C. 12            D.  $36\pi$             E. None of these.
26. Two high school classes took the same test. One class of 20 students made an average of 80%; the other class of 30 students made an average of 70%. The average grade for all students in both classes is
- A. 75%            B. 74%            C. 72%            D. 77%            E. None of these
27. When the circumference of a balloon is increased from 20 inches to 25 inches, the radius is increased by
- A.  $\frac{5}{2\pi}$  inches    B.  $\frac{5}{\pi}$  inches    C. 2.5 inches    D. 5 inches    E. None of these
28. In the rectangle  $ABCD$ , the point  $X$  is located on  $BC$  as shown in the figure. If  $CD = 8$ ,  $AX = 17$ , and  $DX = 10$ , what is the area of  $\triangle ADC$ ?



- A. 108            B. 92            C. 85            D. 84            E. None of these.
29. On a spelling exam, taken by 7 students, 2 scored an A, 3 scored a B, and 2 scored a C. If 3 of these 7 students are chosen at random, what is the probability that none of them scored an A?
- A.  $\frac{1}{7}$             B.  $\frac{2}{7}$             C.  $\frac{3}{7}$             D.  $\frac{4}{7}$             E. None of these.

30. After rationalizing the numerator of  $\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3}}$ , the denominator in simplest form is

- A.  $\sqrt{3}(\sqrt{3} + \sqrt{2})$       B.  $\sqrt{3}(\sqrt{3} - \sqrt{2})$       C.  $3 - \sqrt{6}$   
 D.  $3 + \sqrt{6}$       E. None of these.

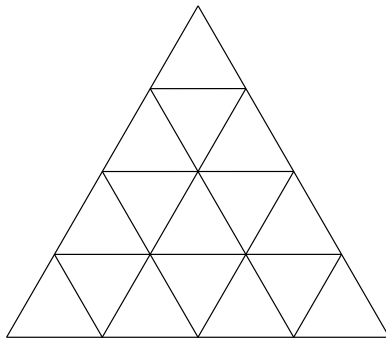
31. If the expression  $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$  has the value  $ab - cd$  for all values of  $a, b, c,$  and  $d,$  then the equation  $\begin{vmatrix} 2x & 1 \\ x & x \end{vmatrix} = 3$  is true for how many values of  $x$ ?

- A. 0      B. 1      C. 2      D.  $\infty$       E. None of these.

32. Given  $x > y$  and  $z \neq 0,$  which of the inequalities below is **not always** correct?

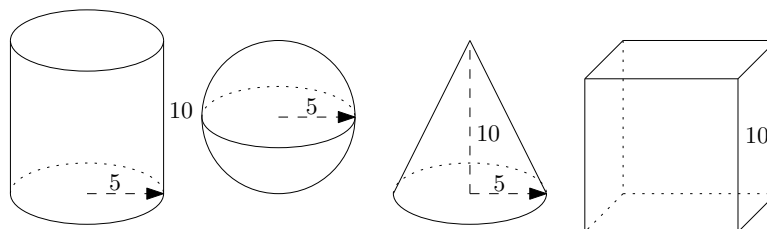
- A.  $x + z > y + z$       B.  $x - z > y - z$       C.  $xz^2 > yz^2$   
 D.  $\frac{x}{z^2} > \frac{y}{z^2}$       E.  $xz > yz$

33. How many triangles are in the following figure?



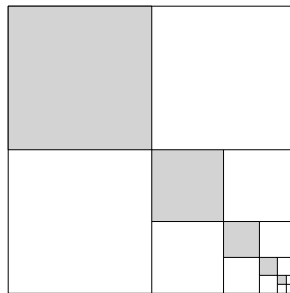
- A. 10      B. 16  
 C. 17      D. 27  
 E. None of these.

34. You are given a cylinder of radius 5 and height 10, a sphere of radius 5, a right circular cone of radius 5 and height 10, and a cube of side length 10 as shown in the figure. Which object has the smallest surface area to volume ratio?



- A. cylinder      B. sphere      C. cone      D. cube      E. my head

35. The front of a box has area 12 square inches, the side has area 8 square inches, and the bottom has area 6 square inches. What is the volume of the box (in cubic inches)?
- A. 576      B. 109      C. 24      D. 9      E. None of these.
36. A tired student wakes up and grabs two socks out of her drawer without looking. If she has 3 identical black socks and 3 identical green socks, what is the probability that she will choose a matching pair?
- A. 0%      B. 10%      C. 20%      D. 30%      E. None of these.
37. A number when divided by 10 leaves a remainder of 9, when divided by 9 leaves a remainder of 8, by 8 leaves a remainder of 7, and so on, down to where, when divided by 2, it leaves a remainder of 1. A possible value for the number is
- A. 59      B. 419      C. 1259      D. 2519      E. None of these.
38. A total of 28 handshakes was exchanged at the conclusion of a party. Assuming that each participant was equally polite toward all the others, the number of people present was
- A. 7      B. 8      C. 14      D. 28      E. None of these.
39. A square is cut into four equal congruent squares and the upper left square is shaded in. The lower right square is then split into four more congruent squares and the upper left square is shaded in. This process is continued forever as demonstrated in the figure. What proportion of the square will eventually be shaded in?



- A.  $\frac{1}{2}$       B.  $\frac{1}{3}$       C.  $\frac{2}{3}$       D.  $\frac{3}{4}$       E. None of these.
40. A group of high school boys and girls are taking a math exam. After an individual finishes, they are free to leave. At first 15 girls leave. At this point there are two boys for each girl left in the room. After this, 45 boys leave. Now there are 5 girls for every boy left in the room. How many girls were in the room to start with?
- A. 40      B. 43      C. 29      D. 50      E. None of these.

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