2014 Marywood Mathematics Contest

Level I

Sponsored by

SEMI-GROUP

The Student Mathematics Club of

Marywood University

and

Mu Alpha Theta

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Directions:

1. This exam consists of 40 questions on 6 pages. Please check to make sure that you have all the pages.

2. No calculator or any other electronic device is allowed on this exam.

3. Allot your time accordingly. This is a 60-minute test. Do not spend too much time on any one problem. If a question seems to be too difficult, make your best possible guess. Your score will be the number of correct responses.

4. On the scantron form provided for you, darken in the space corresponding to the correct answer. Please mark all answers carefully and erase completely when changing an answer. Mark **only one answer** for each question. Only those answers on the answer sheet will be counted.

5. There is a sheet of blank paper on the last page which you can tear off and use as scratch paper. You may also use the back of the pages.

6. NOTE: **In order to ensure uniformity, proctors are NOT allowed to answer any questions pertaining to specific problem content.**

Please do NOT open the test until you are told to do so.

1. How many minutes represent 10% of a full day?

    A. 144   **B.** 80   **C.** 96   **D.** 130   **E.** None of these.

2. If you have 4 choices for the appetizer, 4 choices for the main meal, 4 choices for the dessert, and 4 choices for the drink, how many different choices do you have for your dinner?

    A. 16   **B.** 64   **C.** 256   **D.** 1024   **E.** 2048

3. Solve $3x^2 = 6x + 12$ for x, and the solutions are

    A. 2 and 8   **B.** $-1 \pm \sqrt{5}$ **C.** $1 \pm \sqrt{5}$ **D.** $1 \pm 3\sqrt{5}$ **E.** None of these

4. Evaluate $\sqrt{18} - \sqrt{32} - \sqrt{50}$:

    A. $-\sqrt{2}$ **B.** $-6\sqrt{2}$ **C.** $-13\sqrt{2}$ **D.** $2\sqrt{2}$ **E.** 2

5. Find the x-intercept of $y^{1/2} = 2x - 3xy^2 - 4$.

    A. $(0, 4)$ **B.** $(0, 2)$ **C.** $(2, 0)$ **D.** $(4, 0)$ **E.** $(2, 4)$

6. Simplify $\left(\dfrac{x^3 y^5 z^{-2}}{x^{-2} y^3 z^4} \right)^3$.

    A. $\dfrac{xy^2}{z^6}$ **B.** $\dfrac{x^3 y^6}{z^6}$ **C.** $\dfrac{x^{15} y^6}{z^{18}}$ **D.** $\dfrac{x^3 y^6}{z^{18}}$ **E.** None of these.

7. Find the volume of a sphere with a diameter of 6 cm.

    A. 12π cm^3 **B.** 36π cm^3 **C.** 13.5π cm^3 **D.** 37π cm^3 **E.** None of these.

8. Find the perimeter of the following shape if A is the center of the circular arc from B to C .

    A. $24 + 9\pi/4$

    B. $24 + 9\pi$

    C. 33

    D. $30 + 3\pi$
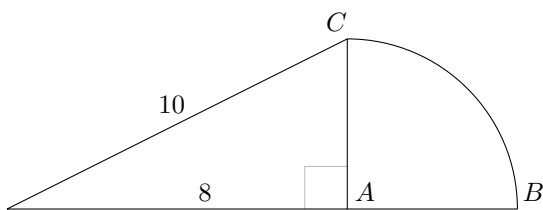
    E. $24 + 3\pi$

9. For what value of k are the lines $2x + 3y = 4k$ and $x - 2ky = 7$ perpendicular?

A. $-3/4$ **B.** $1/6$ **C.** $1/3$ **D.** $1/2$ **E.** $2/3$

10. The area of a circle with diameter d is A. The area of a circle with diameter $2d$ is :

A. $2\pi A$ **B.** $2A$ **C.** $4\pi A$ **D.** $8A$ **E.** None of these.

11. Which of these statements is false?

A. A rectangle is sometimes a rhombus.

B. A rhombus is always a parallelogram.

C. The diagonals of a rectangle are always congruent.

D. The diagonals of a parallelogram always bisect the angles at the vertices.

E. The diagonals of a rhombus always bisect each other.

12. Find the volume of the loaf of bread given that there are 18 slices, each of which are one half-inch thick and have a cross-sectional area of 13 in².

A. 36 in³   **B.** 117 in³   **C.** 234 in³   **D.** 468 in³   **E.** 936 in³

13. A bank teller has 25 more five-dollar bills than ten-dollar bills. The total value of the money is \$200.00. How many five-dollar bills does he have?

A. 28   **B.** 32   **C.** 25   **D.** 20   **E.** None of these.

14. Which of the following sequences of numbers are ranked from greatest to least?

A. $2^2, \pi^\pi, \pi^2, 2^\pi$ **B.** $\pi^\pi, \pi^2, 2^2, 2^\pi$ **C.** $2^\pi, \pi^\pi, \pi^2, 2^2$ **D.** $\pi^\pi, 2^\pi, \pi^2, 2^2$ **E.** None of these.

15. Milk that has 5% butterfat is mixed with milk that has 2% butterfat. How much of each is needed to obtain 60 gallons of milk that has 3% butterfat?

A. 20 gallons of 2% butterfat and 40 gallons of 5% butterfat

B. 40 gallons of 2% butterfat and 20 gallons of 5% butterfat

C. 25 gallons of 2% butterfat and 35 gallons of 5% butterfat

D. 30 gallons of both

E. It cannot be determined with the information given.

16. What quadrilateral has the points $(-3, 6), (-1, -2), (7, -4)$ and $(5, 4)$ in the xy-plane as its vertices?

    A. Rhombus   **B.** Square

    C. Parallelogram but not a rhombus   **D.** Trapezoid but not a parallelogram

    E. Rectangle but not a square.

17. In how many points do the curves $x = y^2$ and $x^2 = (y-1)^2$ intersect?

    A. 1   **B.** 2   **C.** 3   **D.** 4   **E.** 0

18. What is the length of the longest diagonal in a cube with side length 2?

    A. $2\sqrt{3}$ **B.** $2\sqrt{2}$ **C.** $\sqrt{3}$ **D.** 4

19. Peter's age is half that of Corey's. Joe is nine years younger than Corey. In eight years, Peter will be three years older than Joe's current age. What is the sum of their current ages?

    A. 59   **B.** 95   **C.** 61   **D.** 48   **E.** None of these

20. Which of the following is an equation of the line tangent of the circle $x^2 + y^2 = 2$ at the point $(1, 1)$?

    A. $x = 1$ **B.** $y = 1$ **C.** $x + y = 2$ **D.** $x - y = 0$ **E.** $x + y = 0$

21. What is the least possible value of y if (x, y) satisfies the inequality: $|x+4| + |y+5| \le 100$?

    A. -104 **B.** 95   **C.** 96   **D.** -105 **E.** None of these.

22. Jen had an average score of 80 on her first eight quizzes. She had an average score of 82 on her first nine quizzes. What score did she receive on her ninth quiz?

    A. 98   **B.** 80   **C.** 82   **D.** 92   **E.** None of these.

23. At what point in the coordinate plane does the line through the points (0,4) and (16,20) intersect the line $y - x = 5$?

    A. $(1, 5)$ **B.** $(1, 6)$ **C.** $(0, 5)$ **D.** $(6, 1)$ **E.** None of these.

24. Find the sum of all real solutions of $8^{x^2+x+4} = 16^{x^2+2x}$.

    A. 5   **B.** -5 **C.** 12   **D.** -12 **E.** None of these.

25. At a party, each person shakes hands with every other person. There are a total of 5 handshakes. How many people are at the party?

    A. 10   **B.** 11   **C.** 12   **D.** 13   **E.** 14

26. The house numbers in a neighborhood range from 101 to 250 consecutively. If the Home Owner's Association decides to purchage new stickers to replace all the house numbers and each sticker contains exactly one digit, how many stickers with the number 3 should the HOA purchase?

    A. 32   **B.** 33   **C.** 34   **D.** 35   **E.** None of these.

27. How many zeros are at the end of the product of the first fifty positive integers (a.k.a. 50!)?

    A. 10   **B.** 11   **C.** 12   **D.** 13   **E.** None of these.

28. If x and y are positive integers and the average of 17, 7, and x is equal to average of y and 16, then the ratio $x : y$ is

    A. $3 : 2$ **B.** $2 : 3$ **C.** $1 : 1$ **D.** $2 : 5$ **E.** None of these.

29. A fraction m/n is in the lowest terms and its numerator is 4 less than the denominator. If 3 is subtracted from the numerator and 1 is added to the denominator, the resulting fraction is equal to one half. What is $m + n$?

    A. 22   **B.** 25   **C.** 27   **D.** 31   **E.** None of these.

30. For what value of c will the system given below have no solutions?

$$\begin{cases} 2x & + & 6y & = & -2 \\ -3x & + & cy & = & 5 \end{cases}$$

   **A.** 9   **B.** -9 **C.** 4   **D.** -4 **E.** None of these.

31. Compute the value of $\left(1 - \dfrac{3}{7}\right)\left(1 - \dfrac{3}{8}\right)\left(1 - \dfrac{3}{9}\right)\left(1 - \dfrac{3}{10}\right)\cdots\left(1 - \dfrac{3}{20}\right)$.

A. $\dfrac{10}{171}$ **B.** $\dfrac{1}{3}$ **C.** $\dfrac{17}{20}$ **D.** $\dfrac{18}{21}$ **E.** $\dfrac{1}{57}$

32. If the sum of two integers is 5 and the sum of their cubes is 35, what is the sum of their squares?

A. 13 **B.** 21 **C.** 23 **D.** 38 **E.** 57

33. If 100 is divided by a positive integer x, the remainder is 2. What is the remainder when 198 is divided by x?

A. 2 **B.** 3 **C.** 7 **D.** 8 **E.** None of these

34. Suppose x, y, and z are positive integers with $x < y < z$ such that $1/x + 1/y + 1/z = 1$. What is xyz?

A. 18 **B.** 24 **C.** 32 **D.** 36 **E.** None of these

35. What is the quotient when $a^4 - b^4$ is divided by $a - b$?

A. $a^3 - b^3$ **B.** $a^3 + 3a^2b + 3ab^2 + b^5$ **C.** $a^3 + b^3$

D. $a^3 + a^2b + ab^2 + b^3$ **E.** None of these.

36. The integers $x, y, x + y$, and $x - y$ are all positive primes. What is the sum of these four numbers?

A. 7 **B.** 11 **C.** 13 **D.** 17 **E.** None of these.

37. Define the operation $\triangle$ on real numbers by: $a \triangle b = (a + b)/2$. Which of the following is true?

A. $(a\triangle b)\triangle c = a\triangle(b\triangle c)$ **B.** $a + (b\triangle c) = (a + b)\triangle(a + c)$

C. $a\triangle(b + c) = (a\triangle b) + (a\triangle c)$ **D.** $a\triangle 1 = a$

E. None of these is true.

5

38. How many triangles are there in this figure?
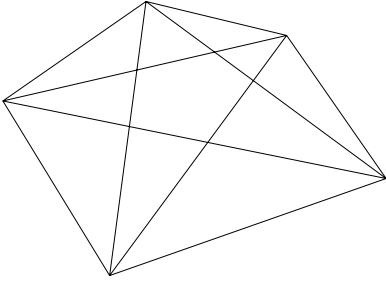
A. 10

B. 15

C. 25

D. 35

E. 45

39. Triangle PQR is equilateral. $QR = 30$ units. B is the midpoint of QA and QA is perpendicular to PR. What is the length of PB?
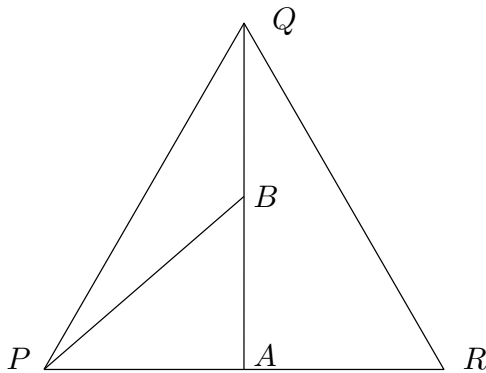
A. $\dfrac{15\sqrt{7}}{2}$

B. $\dfrac{15\sqrt{7}}{4}$

C. $\dfrac{15\sqrt{14}}{2}$

D. $\dfrac{15\sqrt{14}}{4}$

E. None of these.

40. The sum of the digits of 2014 is

A. 0   **B.** 7   **C.** 8   **D.** 9   **E.** None of these.