

2015 Marywood Mathematics Contest

Level I

Sponsored by

iMACS

The Marywood Math and Computer Science Club

Marywood University

and

Mu Alpha Theta

March 21, 2015

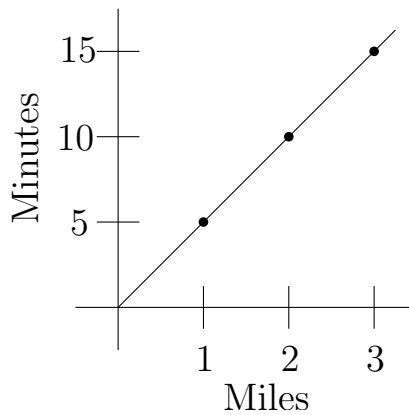
Directions:

1. This exam consists of 40 questions on 6 pages. Please check to make sure that you have all the pages.
2. No calculator or any other electronic device is allowed on this exam.
3. Allot your time accordingly. This is a 60-minute test. Do not spend too much time on any one problem. If a question seems to be too difficult, make your best possible guess. Your score will be the number of correct responses.
4. On the scantron form provided for you, darken in the space corresponding to the correct answer. Please mark all answers carefully and erase completely when changing an answer. Mark **only one answer** for each question. Only those answers on the answer sheet will be counted.
5. There is a sheet of blank paper on the last page which you can tear off and use as scratch paper. You may also use the back of the pages.
6. NOTE: In order to ensure uniformity, proctors are NOT allowed to answer any questions pertaining to specific problem content.

Please do NOT open the test until you are told to do so.

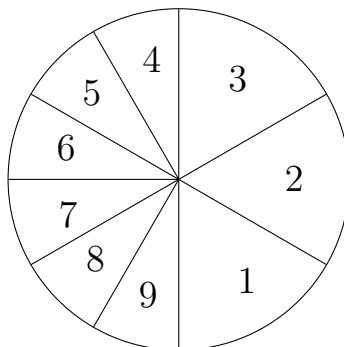
1. $\frac{1}{2} + \frac{1}{3} =$
A. $\frac{1}{5}$ B. $\frac{2}{5}$ C. $\frac{1}{6}$ D. $\frac{4}{5}$ E. $\frac{5}{6}$
2. Solve the equation $11x + 7 = 44$.
A. $x = -3$ B. $x = 7/11$ C. $x = 37/11$ D. $x = 4$ E. $x = 51/11$
3. How many prime numbers are there between 1 and 20?
A. 6 B. 7 C. 8 D. 9 E. None of these.
4. One million divided by one one-hundredth equals
A. 1,000,000,000 B. 100,000,000 C. 10,000,000 D. 100,000 E. 10,000
5. $\left((2^2)^2\right)^2 =$
A. 16 B. 64 C. 128 D. 256 E. None of these.
6. If $x \bowtie y = x^2 - y$, what is $(2 \bowtie 3) \bowtie (3 \bowtie 2)$?
A. -6 B. -4 C. 2 D. 20 E. None of these.
7. $\sqrt{32} - \sqrt{18} =$
A. $\sqrt{14}$ B. $\sqrt{2}$ C. $5\sqrt{2}$ D. $7\sqrt{2}$ E. None of these.
8. Solve $3x^2 = 11x + 42$ for x . The solutions are
A. $\frac{8}{3}$ and 11 B. $-\frac{8}{3}$ and 5 C. 7 and 8 D. $\frac{7}{3}$ and -6 E. None of these.
9. A triangle has sides of length 3, 4, and 5. What is its area?
A. 6 B. 10 C. 7.5 D. 12 E. None of these.
10. $2^{2015} + 2^{2015} =$
A. 2^{2016} B. 4^{2015} C. 2^{4030} D. 4^{2016} E. None of these.

11. Which of the following is a factor of $x^2 + 8x - 65$?
- A. $x + 5$ B. $x - 5$ C. $x - 8$ D. $x + 8$ E. $x - 13$
12. At the movie store, you can rent DVDs for \$1.50 a day, or you can get unlimited rentals for a monthly fee of \$20. At most how many DVDs could you rent at the daily rate before spending more than the cost for a month of unlimited rentals?
- A. 12 B. 13 C. 14 D. 15 E. 16
13. Shauna is out for a leisurely jog in the park. The following graph shows how long she has been running as she passes various mile markers. If she keeps running at this same rate, how far will she have run after 60 minutes?



- A. 10 miles B. 11 miles C. 12 miles D. 13 miles E. None of these.
14. Simplify $\left(\frac{x^7y^3z^{-1}}{x^{-5}yz^8}\right)^{-2} \cdot \left(\frac{3y}{x^2z}\right)^{-3}$.
- A. $\frac{z^{21}}{27x^{17}y^5}$ B. $\frac{z^{21}}{27x^{18}y^5}$ C. $\frac{z^{11}}{27xy^7}$ D. $\frac{z^{21}}{27x^{18}y^7}$ E. None of these.
15. There is a room full of cats and birds. If there are 18 legs in the room and there are as many cats as birds, how many cat legs are in the room?
- A. 3 B. 6 C. 9 D. 12 E. None of these.
16. On average, for every 4 sports cars sold at the local dealership, 7 sedans are sold. The dealership predicts that it will sell 28 sports cars next month. How many sedans does it expect to sell?
- A. 7 B. 32 C. 35 D. 49 E. None of these.

17. A fair circular spinner is divided into two equal halves as shown. The right half is subdivided into three equal sectors and the left half is divided into 6 equal sectors. The sectors are numbered as shown in the figure. If the spinner is spun once, what is the probability of getting an odd number?

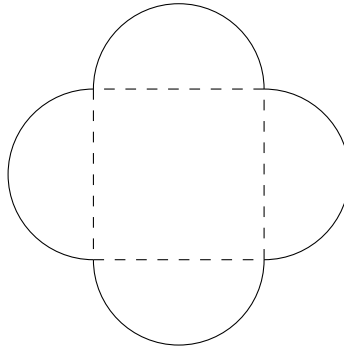


- A. $5/12$ B. $6/12$ C. $7/12$ D. $8/12$ E. None of these.
18. A bank teller has 25 more five-dollar bills than ten-dollar bills. The total value of the money is \$200.00. How many five- and ten-dollar bills does he have in total?
- A. 30 B. 20 C. 25 D. 45 E. None of these.
19. Due to construction delays, it takes Antonia 4 hours to drive from Scranton to Philadelphia which is 100 miles away. How fast must she drive from Philadelphia back to Scranton if she wants to average 50mph for the entire trip?
- A. 50 B. 75 C. 100 D. 125 E. None of these.
20. If 8 cows eat 9 acres of grass in 4 days, how much grass will 2 cows eat in 8 days?
- A. 2.25 acres B. 4.5 acres C. 9 acres D. 12 acres E. 18 acres
21. A box has length 4, width 16, and height 8. How many $2 \times 2 \times 2$ cubes can fit in the box?
- A. 16 B. 32 C. 64 D. 128 E. None of these.
22. A family has two children. What is the probability that both children are girls given that at least one of them is a girl?
- A. $1/4$ B. $1/3$ C. $1/2$ D. 1 E. None of these.

23. Derek has 5 t-shirts, 4 sweaters, 3 pairs of jeans, and two pairs of shoes. If he always wears one t-shirt, one sweater, a pair of jeans, and a pair of shoes, how many different outfit combinations can he wear?

A. 24 B. 30 C. 40 D. 60 E. None of these.

24. A square that is 10cm on each side is decorated by adding semi-circles to each edge. What is the total area of the decorated square?



A. $100 + 50\pi$ B. $100 + 100\pi$ C. $200 + 50\pi$ D. $200 + 100\pi$ E. None of these.

25. The length of a rectangle is increased by 50% and the width is decreased by 50%. What percent of the old area is the new area?

A. 25 B. 50 C. 75 D. 100 E. None of these

26. What is the last digit of 3^{2015} ?

A. 1 B. 3 C. 5 D. 7 E. 9

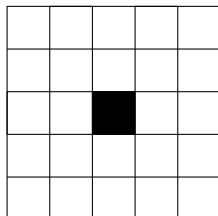
27. Which of these statements is false?

- A. Equilateral triangles are all similar to each other.
- B. An isosceles triangle may have an obtuse angle.
- C. An equiangular triangle is isosceles.
- D. Scalene triangles always have no congruent angles.
- E. An isosceles triangle can not be a right triangle.

28. How many 2 digit positive integers have digits whose product is 16?

A. 3 B. 4 C. 5 D. 6 E. None of these.

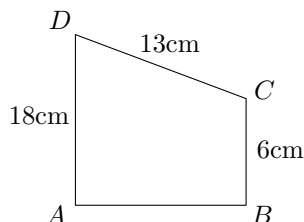
29. The sum of two numbers is 20 and the difference of their squares is 60. What is their product?
- A. -32 B. -16 C. 16 D. 32 E. None of these.
30. Depending on where parentheses are entered, the expression $1 \times 2 + 3 \times 4$ can have a number of different values. How many different values are there?
- A. 2 B. 3 C. 4 D. 5 E. None of these.
31. How many 2 digit numbers have their digits in increasing order? For example 12, but not 21 nor 22.
- A. 34 B. 35 C. 36 D. 37 E. None of these.
32. Which of the following line is tangent to the circle $x^2 + y^2 = 2$ at $(1, 1)$?
- A. $x = 1$ B. $y = 1$ C. $x - y = 0$ D. $x + y = 2$ E. $x + y = 0$
33. How many squares are there in the figure below which include the black square?



- A. 13 B. 14 C. 15 D. 16 E. None of these.
34. During a drive to New York, Celina asks her mom, “Are we there yet?” Her mom responds, “No, but we are $\frac{3}{7}$ of the way there.” Just 24 miles later, Celina asks again, “Are we there yet?” Her mom says, “Not yet, but now we are $\frac{5}{7}$ of the way there.” How many miles have they driven?
- A. 14 B. 24 C. 36 D. 60 E. None of these.
35. The first digit of the decimal expansion of $\frac{1}{7}$ is 1. What is the 2015th digit?
- A. 2 B. 4 C. 7 D. 8 E. None of these.

36. What is the sum of all the **digits** of the integers from 1 to 100? For example, the sum of the digits of the integers from 9 to 11 is $9 + 1 + 0 + 1 + 1 = 12$.
- A. 900 B. 901 C. 902 D. 903 E. None of these.

37. What is the area, in square centimeters, of this trapezoid with right angles at points A and B? (The drawing is not to scale.)



- A. 60 cm^2 B. 90 cm^2 C. 78 cm^2 D. 156 cm^2 E. None of these.
38. For what value of k does the line through the points $(k + 1, 3 + 2k)$ and $(k - 1, 1 - k)$ have slope equal to k ?
- A. -3 B. -2 C. 2 D. 3 E. 4
39. What is the least value of y that may satisfy the inequality: $|x + 20| + |y - 15| \leq 2015$?
- A. -2000 B. 2000 C. -2015 D. 2015 E. None of these.
40. What is the greatest number of factors that a positive integer less than 100 can have?
- A. 3 B. 8 C. 10 D. 12 E. None of these.

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