2012 Marywood Mathematics Contest

Level II

Sponsored by

SEMI-GROUP

The Student Mathematics Club of

Marywood University

March 24, 2012

Directions:

1. This exam consists of 40 questions on 6 pages. Please check to make sure that you have all the pages.

2. No calculator or any other electronic device is allowed on this exam.

3. Allot your time accordingly. This is a 60-minute test. Do not spend too much time on any one problem. If a question seems to be too difficult, make your best possible guess. Your score will be the number of correct responses.

4. On the scantron form provided for you, darken in the space corresponding to the correct answer. Please mark all answers carefully and erase completely when changing an answer. Mark **only one answer** for each question. Only those answers on the answer sheet will be counted.

5. There is a sheet of blank paper on the last page which you can tear off and use as scratch paper. You may also use the back of the pages.

6. NOTE: **In order to ensure uniformity, proctors are NOT allowed to answer any questions pertaining to specific problem content.**

Please do NOT open the test until you are told to do so.

1. $(-3)^{-2} + (-2)^{-3} =$

A. $\dfrac{1}{72}$ **B.** $-\dfrac{1}{72}$ **C.** $\dfrac{17}{72}$ **D.** $-\dfrac{17}{72}$ **E.** None of these.

2. On day one of a long journey, Corey drove 20% of the way to his destination, got tired, pulled over, and slept. The second day he drove 30% of the remaining way to his destination, got tired, pulled over, and slept. If there were 56 miles left after the second day, how many miles did Corey drive the first day?

A. 10 **B.** 15 **C.** 20 **D.** 25 **E.** None of these.

3. Solve the equation $\log_{10}(x + 2) + \log_{10}(x - 3) = \log_{10} 6$, to get $x =$

A. -3. **B.** 3. **C.** -4. **D.** 4. **E.** None of these.

4. Find the sum of all integers from 1 to 2012: $1 + 2 + 3 + \cdots + 2010 + 2011 + 2012 =$

A. 2013×1006 **B.** 2013×1005 **C.** 2012×1006 **D.** 2012×1005 **E.** None of these.

5. If $\sin \theta = -0.8$ and $-\pi/2 < \theta < 0$, what is $\tan \theta$?

A. $3/4$ **B.** $4/3$ **C.** $-3/4$ **D.** $-4/3$ **E.** None of these.

6. A sleepy student wakes up in the morning and needs some socks. She reaches into the drawer to grab a pair, but since the room is dark, she cannot see their colors. She knows that she has 2 pairs of green socks, 3 pairs of purple socks, 4 pairs of aquamarine socks, and 5 pairs of pink socks in the drawer. What is the minimum number of socks she needs to take out of the drawer in order to guarantee that she has at least two socks of the same color?

A. 2 **B.** 3 **C.** 4 **D.** 5 **E.** None of these.

7. Find the radius of a circle circumscribed about a triangle with sides 3, 4, and 5.

A. 1 **B.** 1.5 **C.** 2 **D.** 2.5 **E.** None of these.

8. How many nonreal complex solutions does the equation $x^2 + 2012x + 2013 = 0$ have?

A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** None of these.

9. If $\sin x = \dfrac{1}{3}$, and $0 < x < \pi/2$, what is $\cos(2x)$?

A. $\dfrac{1}{2}$ **B.** $-\dfrac{\sqrt{8}}{3}$ **C.** $\dfrac{\sqrt{8}}{3}$ **D.** $\dfrac{7}{9}$ **E.** None of these.

10. The binary function $*$ is defined by $a * b = \dfrac{a \cdot b - a}{a \cdot b - b}$. What is $(2 * 3) * 2$?

A. -3/2   **B.** -2/3   **C.** 2   **D.** 1/2   **E.** None of these.

11. A palindrome is a number which is same whether it is written forwards or backwards. For example, the number 13931 is a palindrome. How many positive integers less than 1000 are palindromes?

A. 117   **B.** 108   **C.** 99   **D.** 90   **E.** None of these.

12. The sum of the third and fourth terms in a sequence of consecutive integers is 53. The sum of the first five terms of the sequence is

A. 130   **B.** 102   **C.** 99   **D.** 81   **E.** None of these.

13. The function $f(x) = |4\cos(2012x - \pi) - 2|$ has a maximum value of

A. 2   **B.** 3   **C.** 4   **D.** 5   **E.** None of these.

14. Which of the following numbers is least?

A. 6^{100} **B.** 5^{200} **C.** 4^{300} **D.** 3^{400} **E.** 2^{500}

15. Jillian drives to school at an average speed of 75 miles per hour. At what average speed would she have to travel on the return trip in order to average 60 miles per hour for the round trip? (So her parents don't bust her for speeding.)

A. 45 mph   **B.** 50 mph   **C.** 55 mph   **D.** 60 mph   **E.** None of these.

16. Find the intersection point (x_0, y_0) of the lines $y = -3x + 1$ and $2x + 4y = -1$. The value of $x_0 + y_0$ is

A. -2 **B.** -1 **C.** 2   **D.** 1   **E.** None of these.

17. A cube measuring 100 units on each side is painted only on the outside and cut into unit cubes. The number of cubes with paint only on two sides is

**A.** 1000 **B.** 1125 **C.** 1176 **D.** 980 **E.** None of these.

18. $\sqrt{x\sqrt[3]{x\sqrt[4]{x}}} =$

A. $x^{13/24}$. **B.** $x^{15/24}$. **C.** $x^{16/24}$. **D.** $x^{17/24}$. **E.** None of these.

19. Rogelio can mow a yard in 30 minutes. It takes Erin only 20 minutes to mow the same yard. How many minutes would it take them to mow if they worked together using the two mowers?

A. 12 **B.** 50 **C.** 13 **D.** 14 **E.** None of these.

20. In the configuration below consisting of 25 one-by-one squares, how many total squares with horizontal and vertical sides can be formed using the points as vertices?
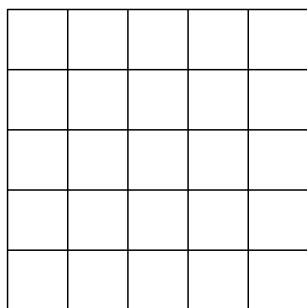
A. 53

B. 54

C. 55

D. 56

E. None of these.

21. In an urn, 5 red marbles and 9 blue marbles are mixed together. If two marbles are drawn at random, what is the probability that one is red and one is blue?

A. 1/2 **B.** 45/91 **C.** 1/14 **D.** 4/45 **E.** None of these.

22. An 8 foot tall sasquatch is standing 20 feet away from a light pole which is 24 feet tall. How long is the beast's shadow?

A. 8 **B.** 11 **C.** 14 **D.** 17 **E.** None of these.

23. A lattice point in the plane is a point both of whose coordinates are integers. How many lattice points (including the endpoints) are there on the line segment joining the points (2, 0) and (16, 203)?

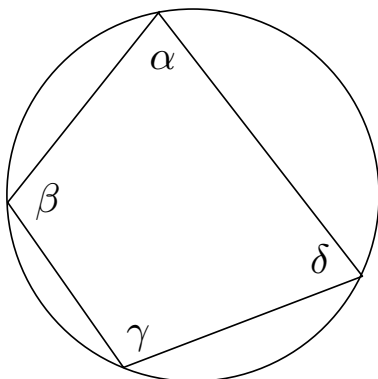**A.** 15 **B.** 8 **C.** 9 **D.** 14 **E.** None of these.

24. Consider the continued fraction

$$x = \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}.$$

The value of x is equal to

A. $\frac{1+\sqrt{5}}{2}$ **B.** $\frac{1-\sqrt{5}}{2}$ **C.** $\frac{-1+\sqrt{5}}{2}$ **D.** $\frac{-1-\sqrt{5}}{2}$ **E.** None of these.

25. The four angles of a quadrilateral inscribed in a circle are α, β, γ, and δ as shown. Which of the following is necessarily true?
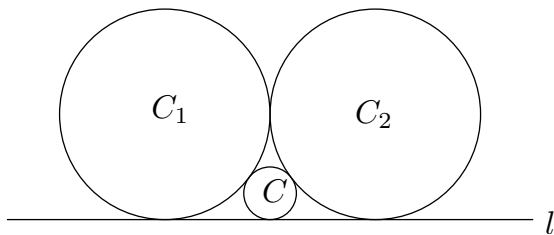

A. $\alpha + \beta + \gamma + \delta = 180°$

B. $\alpha + \beta = 180°$

C. $\beta + \delta = 90°$

D. $\alpha + \gamma = \beta + \delta$

E. None of these.

26. The largest constant C such that $\sin x \geq Cx$ for all x in $[0, \pi/2]$ is

A. 0   **B.** 1/2   **C.** $2/\pi$ **D.** $\pi/2$ **E.** None of these.

27. Circle C is tangent to line l. Two circles C_1 and C_2 of equal radii are each tangent to one another, to C, and to l. If the radius of C is 2, then the radius of C_1 is

A. 6

B. 8

C. 10

D. 12

E. None of these.

28. The unit digit (base 10) of 2^{2012} is

A. 1   **B.** 2   **C.** 4   **D.** 8   **E.** None of these.

4

**29.** For a certain integer n, $5n + 16$ and $8n + 29$ have a common factor larger than one. That common factor is

A. 19   **B.** 17   **C.** 13   **D.** 11   **E.** None of these.

30. For what values of a does the system of equations

$$
\begin{aligned}
x^2 &= y^2 \\
(x - a)^2 + y^2 &= 1
\end{aligned}
$$

have exactly 3 solutions?

A. $a \geq 0$ **B.** $-1 \leq a \leq 1$ **C.** $a = -1, 0, 1$ **D.** $a = -1, 1$ **E.** None of these.

31. Given a square whose sides have length $2a$, find the area of the region bounded by the 4 semi-circles which are in the interior of the square and have the four sides of the square as diameters.
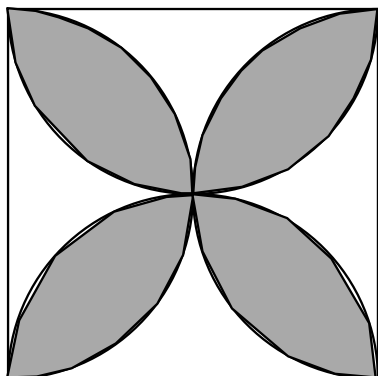
A. $(2\pi - 4)a^2$

B. $\dfrac{\pi a^2}{8}$

C. $\dfrac{(8 - \pi)a^2}{4}$

D. $2a^2$

E. None of these.

32. The sum of two integers is S. Two digits in one of the integers are interchanged and a new sum T is produced. The difference $S - T$ is necessarily divisible by

A. 9   **B.** 7   **C.** 5   **D.** 10   **E.** None of these.

33. Let $X = 2012 + \dfrac{1}{2012}$, $Y = 2012 + \dfrac{1}{2012 + \dfrac{1}{2012}}$, and $Z = 2012 + \dfrac{1}{2012 + \dfrac{1}{2012 + \dfrac{1}{2012}}}$

The numbers X, Y, and Z arranged in increasing order are

A. X, Y, Z **B.** Z, X, Y **C.** Y, Z, X **D.** Z, Y, X **E.** None of these.

5

34. Let a and b be the two roots of the equation $x^2 - x - 1 = 0$. What is $a^2 + b^2$?

A. 2 **B.** 4 **C.** 6 **D.** 8 **E.** None of these.

35. If a fair **four sided** die is tossed 3 times (with possible outcomes of 1, 2, 3, and 4), what is the probability that one will observe at least two 1's consecutively OR at least two 3's consecutively?

A. 1/16 **B.** 1/8 **C.** 3/16 **D.** 1/4 **E.** None of these.

36. A certain function f satisfies $f(x) = 2f(6 - x) - x$ for all real numbers x. The value of $f(1)$ is

A. -9 **B.** 1 **C.** 2 **D.** 3 **E.** None of these.

37. There are 120 five-digit numbers that can be formed by permuting 1, 2, 3, 4, and 5, such as
$$12345, 12354, 21435, \ldots, 54321.$$

The sum of all these numbers is

A. 2,876,540 **B.** 3,999,960 **C.** 4,969,960 **D.** 5,600,610 **E.** 6,975,640

38. In quadrilateral $ABCD$, $AB = 2$, $BC = CD = 4$, $DA = 5$, and the opposite angles A and C are congruent. The length of the diagonal BD is
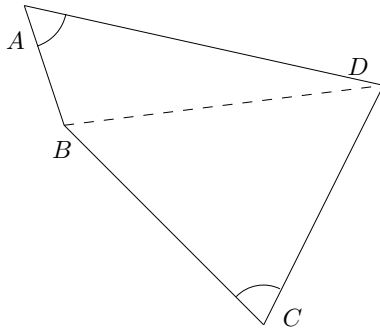
A. $\frac{2}{3}\sqrt{30}$

B. 5

C. $2\sqrt{6}$

D. $6\sqrt{2}$

E. None of these.

39. If $\tan x = \dfrac{\sin 10° + \sin 40°}{\cos 10° + \cos 40°}$, and x is between $0°$ and $90°$, then x is

A. 20° **B.** 30° **C.** 22.5° **D.** 25° **E.** 50°

40. When $x^{100} - 2x^{99} + 4$ is divided by $x^2 - 3x + 2$, the remainder is

A. $x + 1$ **B.** $2x + 1$ **C.** $x - 1$ **D.** $x + 2$ **E.** $3x - 2$

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