2013 Marywood Mathematics Contest

Level II

Sponsored by

SEMI-GROUP

The Student Mathematics Club of

Marywood University

March 16, 2013

Directions:

1. This exam consists of 40 questions on 6 pages. Please check to make sure that you have all the pages.

2. No calculator or any other electronic device is allowed on this exam.

3. Allot your time accordingly. This is a 60-minute test. Do not spend too much time on any one problem. If a question seems to be too difficult, make your best possible guess. Your score will be the number of correct responses.

4. On the scantron form provided for you, darken in the space corresponding to the correct answer. Please mark all answers carefully and erase completely when changing an answer. Mark **only one answer** for each question. Only those answers on the answer sheet will be counted.

5. There is a sheet of blank paper on the last page which you can tear off and use as scratch paper. You may also use the back of the pages.

6. NOTE: **In order to ensure uniformity, proctors are NOT allowed to answer any questions pertaining to specific problem content.**

Please do NOT open the test until you are told to do so.

1. $2013^0 + 2013^{-1} =$

A. -2013 **B.** $-1/2013$ **C.** $1/2013$ **D.** $2012/2013$ **E.** $2014/2013$

2. $(x-1)^2 =$

A. $x^2 - 1$ **B.** $x^2 + 1$ **C.** $x^2 - 2x - 1$ **D.** $x^2 - 2x + 1$ **E.** None of these.

3. Thomas made 12 shots in a basketball game, which is approximately 52% of his attempted shots. How many shots did he attempt?

A. 6 **B.** 7 **C.** 24 **D.** 23 **E.** None of these.

4. When a pair of fair dice are rolled, what is the probability that the sum of the two numbers represented by the dice is 10?

A. $\frac{1}{9}$ **B.** $\frac{1}{10}$ **C.** $\frac{1}{11}$ **D.** $\frac{1}{12}$ **E.** None of these.

5. $3^{2013} + 3^{2013} =$

A. 3^{4026} **B.** 3^{2014} **C.** 2×3^{2013} **D.** 6^{2013} **E.** None of these.

6. Notice that $2013 = 3 \times 11 \times 61$, how many distinct positive factors does 2013 have?

A. 3 **B.** 6 **C.** 7 **D.** 8 **E.** 9

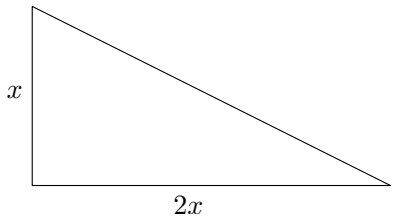
7. If $f(x) = x^{2013} + x + 1$, what is $f(-1)$?

A. 1 **B.** -1 **C.** 2012 **D.** 2013 **E.** 2014

8. The operation $\diamond$ is defined as $a \diamond b = a^2 + ab + b^2$. Find $3 \diamond 2$.

A. 20 **B.** 19 **C.** 13 **D.** 12 **E.** None of these.

9. If one leg of a right trangle is twice the length of the other leg, what is the ratio between the hypotenuse and the shorter leg?



A. $\sqrt{3} : 1$ **B.** $\sqrt{3} : \sqrt{2}$

C. $\sqrt{5} : 1$ **D.** $\sqrt{5} : \sqrt{2}$

E. $5 : 1$

10. Given two points $A(4, 12)$ and $B(12, 20)$, find the point Q on $\overline{AB}$ such that $|AQ| = 3|BQ|$.

A. $(8, 16)$ **B.** $(10, 18)$ **C.** $(6, 14)$ **D.** $(18, 10)$ **E.** None of these.

11. The sum of the two roots of the equation $x^2 + 5x - 1 = 0$ is

A. -5 **B.** -1 **C.** 5 **D.** 1 **E.** None of these.

12. If all of the following have the same area, which one has the **largest** perimeter/circumference?

A. An equilateral triangle **B.** A square **C.** A regular pentagon

D. A regular hexagon **E.** A circle

13. The units digit of 2013^{2012} is

A. 1 **B.** 3 **C.** 7 **D.** 9 **E.** None of these.

14. Professor Johnson lives 10 miles from the university where he works. His morning commute to work takes 30 minutes, while his evening commute home only takes 20 minutes. What is Professor Johnson's combined average speed for his commute each day?

A. 12 mph **B.** 24 mph **C.** 25 mph **D.** 26 mph **E.** None of these.

15. The point $P(-3, 4)$ is on the circle $x^2 + y^2 = 25$. If a tangent line of the circle is drawn through P, where would it intersect with the x-axis?

A. $\left(-\frac{25}{3}, 0\right)$ **B.** $\left(\frac{25}{3}, 0\right)$ **C.** $\left(-\frac{25}{4}, 0\right)$ **D.** $\left(\frac{25}{4}, 0\right)$ **E.** $\left(0, \frac{25}{4}\right)$

16. How many positive integers less than 2013 would make $1^n + 2^n + 3^n$ divisible by 4?

A. 0 **B.** 1005 **C.** 1006 **D.** 2010 **E.** 2011

17. The edge of a cube is of length 2 in. What is the distance from the center point on one face to a vertex on the opposite face?

A. $\sqrt{8}$ in **B.** $\sqrt{7}$ in **C.** $\sqrt{6}$ in **D.** $\sqrt{5}$ in **E.** None of these.

18. A rectangle has a perimeter of 40 cm. If the length and width are each increased by 2 cm, by how much will the total area increase?

A. 40 cm^2 **B.** 44 cm^2 **C.** 80 cm^2 **D.** 96 cm^2

E. It depends on the exact dimension of the rectangle.

19. If $\frac{1}{x} + \frac{1}{2x} = 6$, what is the denominator of x when it is written in the simplest fraction form?

A. 2 **B.** 3 **C.** 4 **D.** 18 **E.** None of these.

20. A regular polygon with n vertices has more than 15 diagonals. What is the smallest possible value of n? (A diagonal of a polygon is defined as the line segment between any two non-adjacent vertices.)

A. 9 **B.** 8 **C.** 7 **D.** 6 **E.** None of these.

21. In how many ways can the number 10 be written as the sum of three **distinct** positive integers? Note that the order of the three integers is insignificant. For example, 1+2+7 and 2+7+1 are considered the same.

A. 4 **B.** 5 **C.** 6 **D.** 7 **E.** 8

22. The pages in a book are numbered from 1 to 314. How many pages have the digit 3 in its page number?

A. 68 **B.** 69 **C.** 71 **D.** 72 **E.** None of these.

23. If $\sin\theta = \frac{1}{10}$, and $0^\circ < \theta < 90^\circ$, what is $\tan\theta$?

A. $\frac{1}{\sqrt{99}}$ **B.** $\frac{1}{\sqrt{9}}$ **C.** $\frac{10}{\sqrt{99}}$ **D.** $\sqrt{99}$ **E.** None of these.

24. If x and y are positive real numbers such that $xy = 12$, what is the minimum possible value of $x^2 + y^2$?

A. 40 **B.** 25 **C.** 24 **D.** 20 **E.** 12

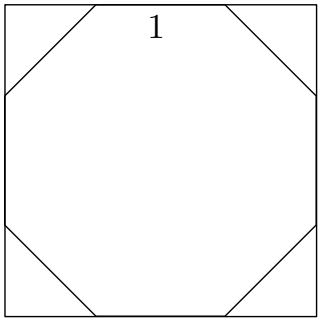
25. If $f(x) = x - 3$, $g(x) = x^2$, then $f \circ g(x) = f(g(x)) =$

A. $x^2 - 3$ **B.** $(x-3)^2$ **C.** $x^2 - 9$ **D.** $x - 9$ **E.** None of these.

26. $\dfrac{1+2+3+\cdots+2012}{1+2+3+\cdots+2012+2013} =$

A. $\dfrac{1}{2013}$ **B.** $\dfrac{2012}{2013}$ **C.** $\dfrac{2012}{2014}$ **D.** $\dfrac{2013}{2014}$ **E.** None of these.

27. The four corners of a square are cut and a regular octagon is formed. If the resulting octagon has side length equal to 1 cm, what is the length of a side in the original square?



A. $\dfrac{\sqrt{2}}{2}$ **B.** $1+\dfrac{\sqrt{2}}{2}$

C. $1+\sqrt{2}$ **D.** $\sqrt{2}$

E. 3

28. How many digits are in the number $4^5 \cdot 5^{10}$?

A. 8 **B.** 9 **C.** 10 **D.** 11 **E.** 12

29. What time was it 2013 minutes after the beginning of January 1, 2013?

A. January 1 at 9:33PM **B.** January 1 at 11:53PM **C.** January 2 at 3:13AM

D. January 2 at 9:33AM **E.** January 2 at 6:03PM

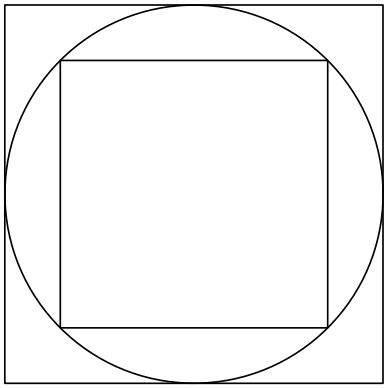
30. Allie, Bobby, Cathy and Dan are seated at random around a square table with one person at each side. What is the probability that Allie and Dan are seated across from each other on opposite sides of the table?

A. $\dfrac{1}{2}$ **B.** $\dfrac{1}{3}$ **C.** $\dfrac{2}{3}$ **D.** $\dfrac{1}{4}$ **E.** $\dfrac{3}{4}$

31. In how many ways can 2015 be written as the sum of two positive **prime** numbers?

A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** 4

32. A circle is inscribed in a square and circumscribed about another square. Which of the following **best approximates** to the ratio of the area between the circle and the smaller square to the area between the two squares?

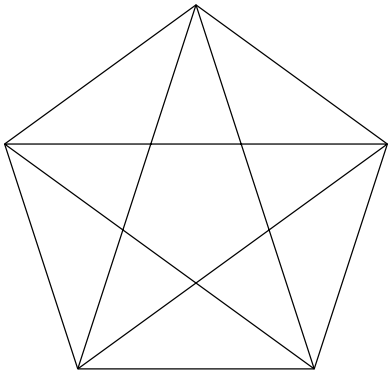


A. 1/2 **B.** 1

C. 3/2 **D.** 2

E. 5/2

33. How many triangles are there in the figure below?



A. 25

B. 30

C. 35

D. 40

E. None of these.

34. $\dfrac{1}{1\cdot 2}+\dfrac{1}{2\cdot 3}+\dfrac{1}{3\cdot 4}+\cdots+\dfrac{1}{2012\cdot 2013}=$

A. $\dfrac{2012}{2013}$ **B.** $\dfrac{2014}{2013}$ **C.** $\dfrac{2013}{2012}$ **D.** 1 **E.** None of these.

35. Bag A contains 10 chips labeled $1, 3, 5, \cdots, 19$, and bag B contains 10 chips labeled $2, 4, 6, \cdots, 20$. If one chip is selected from each bag, how many **different** values are possible as the **sum** of the two numbers on the two chips selected?

A. 18 **B.** 19 **C.** 20 **D.** 39 **E.** 40

36. If an equilateral triangle and a regular hexagon have the same area, what is the ratio between their perimeters?

A. $1:1$ **B.** $\sqrt{3}:6$ **C.** $6:1$ **D.** $\sqrt{6}:1$ **E.** $3:\sqrt{6}$

37. If x and y satisfy the system of equations $\begin{cases} 20x + 13y = 20 \\ 20x - 12y = 15 \end{cases}$, what is the value of $x + y$?

A. 1.07 **B.** 0.67 **C.** 0.2 **D.** 0.93 **E.** None of these.

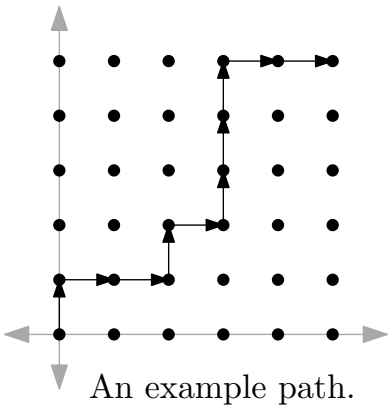
38. If $\sqrt{\sqrt{\sqrt{M}}} = 3$, then $\sqrt{M\sqrt{M\sqrt{M}}} =$

A. 3 **B.** 27 **C.** 81 **D.** 729 **E.** None of these.

39. The three sides of $\triangle ABC$ are all integers in length, and $|AB| = 7$, $|AC| = 12$. If the maximum possible value of $|BC|$ is x, and the minimum possible value of $|BC|$ is y, what is the sum $x + y$?

A. 18 **B.** 19 **C.** 24 **D.** 25 **E.** 26

40. If a point is only allowed to move to the right or up from one lattice point to another. How many different paths are there from $(0, 0)$ to $(5, 5)$? Note: A lattice point is one with integer coordinates.



An example path.

A. 5
B. 10
C. 126
D. 252
E. None of these.

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