2014 Marywood Mathematics Contest

Level II

Sponsored by

SEMI-GROUP

The Student Mathematics Club of

Marywood University

and

Mu Alpha Theta

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Directions:

1. This exam consists of 40 questions on 6 pages. Please check to make sure that you have all the pages.

2. No calculator or any other electronic device is allowed on this exam.

3. Allot your time accordingly. This is a 60-minute test. Do not spend too much time on any one problem. If a question seems to be too difficult, make your best possible guess. Your score will be the number of correct responses.

4. On the scantron form provided for you, darken in the space corresponding to the correct answer. Please mark all answers carefully and erase completely when changing an answer. Mark **only one answer** for each question. Only those answers on the answer sheet will be counted.

5. There is a sheet of blank paper on the last page which you can tear off and use as scratch paper. You may also use the back of the pages.

6. NOTE: **In order to ensure uniformity, proctors are NOT allowed to answer any questions pertaining to specific problem content.**

Please do NOT open the test until you are told to do so.

1. What is the maximum amount of time you may spend on any one problem on this test?

    A. 1 minute   **B.** 1.5 minutes **C.** 2 minutes   **D.** 5 minutes   **E.** None of these.

2. $2014^0 + 0^{2014} =$

    A. 0   **B.** 1   **C.** 2   **D.** 2014   **E.** 2015

3. The value of $\arctan(\sqrt{3})$ is

    A. $\pi/6$ **B.** $5\pi/6$ **C.** $\pi/3$ **D.** $2\pi/3$ **E.** None of these.

4. If $x \heartsuit y = x^2 - y$, what is $(2\heartsuit 3)\heartsuit 4$?

    A. 3   **B.** 9   **C.** -3 **D.** -9 **E.** None of these.

5. If $25^x = 125^{x-1}$, what is the value of x?

    A. 7   **B.** 4   **C.** $\dfrac{1}{2}$ **D.** 3   **E.** None of these.

6. For what value of c will the points $(c, 1), (1, 5), (3, 2)$ be collinear?

    A. $\dfrac{-11}{3}$ **B.** $\dfrac{5}{3}$ **C.** $\dfrac{-5}{3}$ **D.** $\dfrac{11}{3}$ **E.** None of these.

7. The minimum value of the quadratic function $f(x) = 2x^2 - 8x + 1$ is

    A. -7 **B.** 7   **C.** -9 **D.** 9   **E.** None of these.

8. If r and s are the solutions of the quadratic equation $x^2 + 3x + 1 = 0$ and $r > s$, what is $r - s$?

    A. 3   **B.** $\sqrt{5}$ **C.** $\dfrac{3}{2}$ **D.** $\dfrac{\sqrt{5}}{2}$ **E.** None of these.

9. What is the height BD of the right triangle $\triangle ABC$ if $AB = 3$, $BC = 4$, and $\angle B = 90°$?
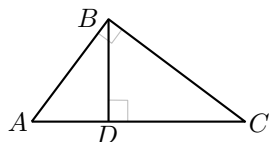

    A. $\dfrac{12}{5}$ **B.** $\dfrac{6}{5}$ **C.** $\dfrac{5}{12}$ **D.** $\dfrac{5}{6}$ **E.** None of these.

10. You decide to play a game of chance with your friend. You both roll two fair six sided dice. If your friend rolls a larger number than you, your friend wins. Otherwise, you win. What is the probability that you will win?

A. $\dfrac{1}{2}$ **B.** $\dfrac{2}{3}$ **C.** $\dfrac{5}{9}$ **D.** $\dfrac{5}{12}$ **E.** $\dfrac{7}{12}$

11. How many lines from one vertex of the hexagon to another can be drawn that lie entirely within the hexagon?
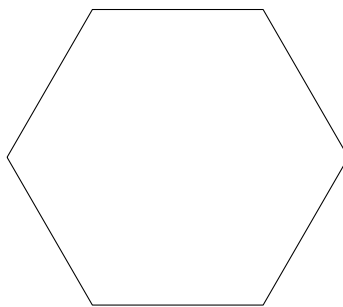
A. 3   **B.** 9   **C.** 12   **D.** 15   **E.** 18

12. If $0 < \theta < \pi/2$, and $\sin\theta = 0.6$, what is $\sin(2\theta)$?

A. 0.48   **B.** 0.8   **C.** 0.96   **D.** 1.2   **E.** None of these.

13. Tom and Jerry are at a meeting with 8 other people. If the 10 seats at the round table are randomly assigned to the attendees, what is the probability that Tom and Jerry will be sitting next to each other?

A. $\dfrac{1}{10}$ **B.** $\dfrac{1}{9}$ **C.** $\dfrac{1}{8}$ **D.** $\dfrac{2}{7}$ **E.** None of these.

14. A Norman window has a rectangular base which is 4 ft long by 5 ft high .and a semicircular top. What is area of the whole window?

A. $20 + 4\pi$ ft^2 **B.** $20 + 8\pi$ ft^2 **C.** $10 + 2\pi$ ft^2 **D.** $20 + 2\pi$ ft^2 **E.** None of these.

15. A family has two children with at least one boy. What is the probability that both are boys?

A. $\dfrac{1}{3}$ **B.** $\dfrac{1}{2}$ **C.** $\dfrac{2}{3}$ **D.** 1   **E.** None of these.

2

16. You drop a few hundred quarters on a table and notice that none of them are overlapping each other and all are laying flat on the table. What is the maximum number of other quarters that any one quarter can be touching?

A. 3   **B.** 4   **C.** 5   **D.** 6   **E.** 7

17. Mr. Speedo averaged 6 miles per hour when going up a hill, but when he bikes back down the hill, his average speed was 9 miles per hour. What was Mr. Speedo's average speed for the roundtrip?

A. 7.0 mph   **B.** 7.2 mph   **C.** 7.5 mph   **D.** 7.8 mph   **E.** None of these.

18. If $(1+x)^{2014}$ is expanded and simplified, how many terms are in the resulting polynomial?

A. 2013   **B.** 2014   **C.** 2015   **D.** 2016   **E.** 2017

19. The students in a calculus class took a midterm exam and the class mean was 78. One student with a score of 45 decided to withdraw from the class, and the mean for the rest of the class went up to 81. How many students were in the class originally?

A. 10   **B.** 11   **C.** 12   **D.** 13   **E.** None of these.

20. If $f(x) = 0.25 \ln\left(\sqrt{4-x}\right)$, then $f^{-1}(x) =$

A. $4 - e^{4x^2}$ **B.** $4 - e^{16x^2}$ **C.** $4 - e^{16x}$ **D.** $4 - e^{8x^2}$ **E.** $4 - e^{8x}$

21. A palindrome is a number that can be read the same forwards as backwards. For example, 127,721 is a palindrome. How many palindromes are there between 10 and 1,000?

A. 98   **B.** 99   **C.** 100   **D.** 101   **E.** 102

22. Cerberus, the three headed dog, has just had a litter of (three headed) puppies!!! Hades, Cerberus' owner, is so excited that he invites some of his good friends from Mount Olympus to see how cute the new pups are. While looking at his guests admire the puppies, Hades notices between his guests and the puppies, he can see 22 heads and 34 feet. How many puppies did Cerberus have? (Just like normal dogs, you may assume that the puppies have 4 legs. The gods of Olympus, being rather human-like in their appearance, all have 1 head and 2 legs - of course.)

A. 3   **B.** 4   **C.** 5   **D.** 6   **E.** 7

23. The remainder of $f(x) = x^{2014} + 2014x^2 + 2014x + 1$ divided by $g(x) = x - 1$ is

    A. 2012   **B.** 2013   **C.** 2014   **D.** 2015   **E.** None of these.

• The following information is used for problems 24 and 25.

         AND   **OR**   **NOT**
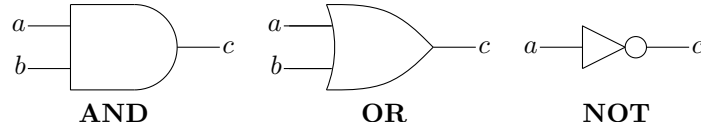
In order to read a circuit diagram you need to know that for an **AND** gate, the output c is 1 exactly when both of the inputs a and b are 1, otherwise it is 0. On the other hand, for an **OR** gate, the output c is 0 exactly when both of the inputs a and b are 0, otherwise it is 1. Finally, for a **NOT** gate, $c = 1 - a$. It is also important to know that the values for a, b, and c can only be 0 or 1. No other value is permitted.

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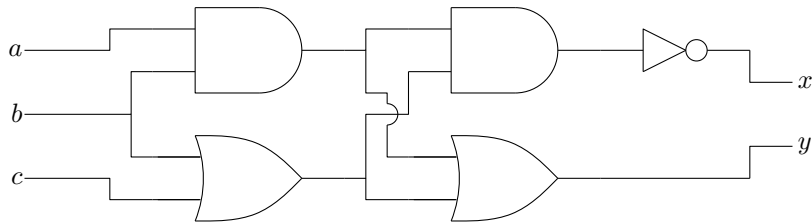
24. If $a = 1$, $b = 0$, and $c = 1$, what is the value of x and y?

    A. $x = 1$, $y = 1$ **B.** $x = 1$, $y = 0$ **C.** $x = 0$, $y = 1$ **D.** $x = 0$, $y = 0$ **E.** None of these.

25. For the above circuit diagram, how may different inputs a, b, and c give an output of $x = 1$ and $y = 0$?

    A. 2   **B.** 3   **C.** 4   **D.** 5   **E.** 6

26. If x is a real number such that $2^x = 2014$, and n is the closest integer to x, then how many digits does $(n - 1)^{n+1}$ have?

    A. 10   **B.** 11   **C.** 12   **D.** 13   **E.** 14

27. What is the base 10 value of the base 5 number $(2014)_5$?

    A. 259   **B.** 279   **C.** 1259   **D.** 1279   **E.** 1007

28. What is the value of $1 + 4 + 7 + 10 + \cdots + 2014$?

A. 2015×336 **B.** 2015×335 **C.** 2015×672 **D.** 2015×670 **E.** None of these.

29. If you cut out the shape below, fold along the solid lines and tape it all together, you will get a solid. How many

A. 10 vertices and 14 edges   **D.** 9 vertices and 9 edges

B. 18 vertices and 18 edges

C. 6 vertices and 14 edges   **E.** 6 vertices and 9 edges

30. A right circular cylinder is of height 5 cm. The area of the top and bottom together equals the area of its side. What is the volume of the cylinder?

A. 25π cm^3 **B.** 125π cm^3 **C.** 250π cm^3 **D.** 50π cm^3 **E.** None of these.

31. What is the smallest positive integer ≥ 100 that has exactly 5 positive divisors?

A. 243   **B.** 256   **C.** 343   **D.** 625   **E.** None of these.

32. A circle is inscribed in a triangle with side lengths 10, 13, 13. What is the radius of the circle?

A. $\dfrac{10}{3}$ **B.** $\dfrac{3}{5}$ **C.** $\dfrac{5}{3}$ **D.** $\dfrac{3}{10}$ **E.** None of these.

33. What is the area of the feasible region defined by the inequalities $x + y \leq 1$, $x - y \leq 1$, $-x - y \leq 1$, and $-x + y \leq 1$?

A. $\dfrac{\sqrt{2}}{2}$ **B.** 1   **C.** $\sqrt{2}$ **D.** 2   **E.** None of these.

34. Let $x = \sqrt{8 - 2\sqrt{8 - 2\sqrt{8 - \cdots}}}$. Then x is also equal to which of the following?

A. 8   **B.** $4\sqrt{2}$ **C.** 4   **D.** $2\sqrt{2}$ **E.** 2

35. You are bored in English class and so you start to fill in the cartesian plan as show in the diagram.

```
15 16 | 5  6
14  4 | 1  7
------+------
13  3 | 2  8
12 11 | 10 9
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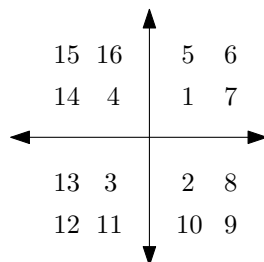
In which quadrant will the number 2014 appear?

A. I **B.** II **C.** III **D.** IV **E.** V

36. $\sin\left(\dfrac{\pi}{3}\right) + \sin\left(\dfrac{2\pi}{3}\right) + \sin\left(\dfrac{3\pi}{3}\right) + \cdots + \sin\left(\dfrac{2014\pi}{3}\right) =$

A. 0 **B.** 1 **C.** $\sqrt{3}$ **D.** $\dfrac{1}{2}$ **E.** $\dfrac{\sqrt{3}}{2}$

37. Given $i = \sqrt{-1}$, evaluate $\displaystyle\sum_{k=0}^{2014} i^k$.

A. $-i$ **B.** -1 **C.** 0 **D.** i **E.** 1

38. Let a, b, c, d be randomly assigned values from the set $\{2, 4, 5, 6\}$ with no repetitions. What is the maximum possible value of the sum of the four products in the table below?

$*$	a	b
c	ac	bc
d	ad	bd

A. 62 **B.** 66 **C.** 72 **D.** 74 **E.** 78

39. Find a positive value of k such that $x^2 + kx = k - 3$ has exactly one real solution.

A. $k = 1$ **B.** $k = 2$ **C.** $k = 3$ **D.** $k = 4$ **E.** $k = 5$

40. The answer to this problem is the letter whose place in the alphabet is equal to the remainder when 38 is divided by 7.

A. A **B.** B **C.** C **D.** D **E.** E

6