

2015 Marywood Mathematics Contest

Level II

Sponsored by

iMACS

The Marywood Math and Computer Science Club

Marywood University

and

Mu Alpha Theta

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Directions:

1. This exam consists of 40 questions on 6 pages. Please check to make sure that you have all the pages.
2. No calculator or any other electronic device is allowed on this exam.
3. Allot your time accordingly. This is a 60-minute test. Do not spend too much time on any one problem. If a question seems to be too difficult, make your best possible guess. Your score will be the number of correct responses.
4. On the scantron form provided for you, darken in the space corresponding to the correct answer. Please mark all answers carefully and erase completely when changing an answer. Mark **only one answer** for each question. Only those answers on the answer sheet will be counted.
5. There is a sheet of blank paper on the last page which you can tear off and use as scratch paper. You may also use the back of the pages.
6. NOTE: In order to ensure uniformity, proctors are NOT allowed to answer any questions pertaining to specific problem content.

Please do NOT open the test until you are told to do so.

1. $\frac{x^2 - y^2}{x - y} =$
A. $x - y$ B. $x + y$ C. $y - x$ D. $x^3 - y^3$ E. None of these.
2. A right circular cylinder has base radius 6 cm, and height 10 cm. What is the volume of the cylinder?
A. 360 cm^3 B. $120\pi \text{ cm}^3$ C. $600\pi \text{ cm}^3$ D. $360\pi \text{ cm}^3$ E. None of these.
3. If $\sqrt{1} + \sqrt{9} = \sqrt{x}$, then $x =$
A. 10 B. 4 C. 16 D. 100 E. None of these.
4. $x^2 + 6x - 16 =$
A. $(x - 2)(x - 8)$ B. $(x - 2)(x + 8)$ C. $(x + 2)(x - 8)$
D. $(x + 4)(x - 4)$ E. $(x - 4)(x - 4)$
5. If $x \searrow y = (x - 1) \times y$, what is $5 \searrow (3 \searrow 4)$?
A. 32 B. 60 C. 40 D. 35 E. 36
6. The lines $3x + y = 2$ and $x + 3y = 2$
A. are parallel. B. are perpendicular.
C. intersect in Quadrant I D. intersect in Quadrant III
E. None of these.
7. $\arccos(-1/2) =$
A. $\pi/6$ B. $\pi/3$ C. $-\pi/3$ D. $2\pi/3$ E. $5\pi/6$
8. If the point $(2, 5)$ is reflected across the x -axis and then across the line $y = x$, the new coordinate of the point will be
A. $(2, -5)$ B. $(-5, 2)$ C. $(5, 2)$ D. $(5, -2)$ E. $(-5, -2)$

9. $(a + b)^3 =$

A. $a^3 + b^3$

B. $a^3 + 2ab + b^3$

C. $a^3 + 3ab + b^3$

D. $a^3 + 3a^2b + 3ab^2 + b^3$

E. None of these.

10. The U.S. government hit its current debt ceiling this past week, which is over \$18 trillion. The current US population is over 320 million. Which of the following is closest to the current US national debt per capita?

A. \$600000

B. \$60000

C. \$6000

D. \$600

E. \$60

11. The ratio of the angles in a triangle is 7 : 10 : 12. What is the average of the three angles in degree measure?

A. 30°

B. 45°

C. 50°

D. 70°

E. None of these.

12. A winter coat originally priced at \$200 was marked down for a 40% off clearance sale at a department store. Sally has another coupon that gives her another discount of 30% from the current price. How much will the coat cost Sally (before tax, if any) if she buys it?

A. \$60

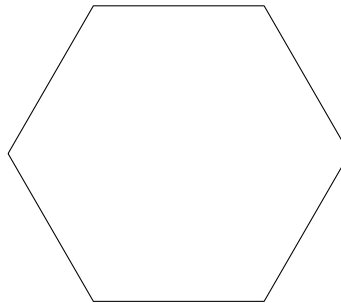
B. \$84

C. \$80

D. \$92

E. None of these.

13. The regular hexagon has side length 10, what is its area?



A. $100\sqrt{3}$

B. $25\sqrt{3}$

C. $300\sqrt{3}$

D. $150\sqrt{3}$

E. None of these.

14. If x and y are positive integers, and the mean of 6, 9, and x is equal to the mean of 10 and y . What is the smallest possible value of $x^2 + y^2$?

A. 5

B. 13

C. 52

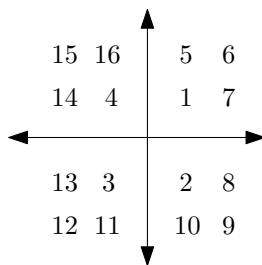
D. 11

E. None of these.

15. Jennifer wants to order a custom license plate with only 4 digits and no letters. Her state has a rule that such a license plate cannot contain the digits 0 or 1, to avoid confusion with the letters O and I. How many such custom license plates are possible?
- A. 10000 B. 4000 C. 40320 D. 4096 E. None of these.
16. Solve the inequality $|3x + 12| < 9$, and the solution in interval notation is:
- A. $(-7, -1)$ B. $(\infty, -7) \cup (-1, \infty)$ C. $(1, 7)$
D. $(-1, 7)$ E. None of these.
17. If $f(x) = x + 1$, what is $f(f(f \cdots f(x) \cdots))$, where the function f is applied 2015 times to x ?
- A. $2014x + 1$ B. $2015x + 1$ C. $2014x + 2015$ D. $x + 2014$ E. $x + 2015$
18. The repeating decimal number $0.2777777777777777 \dots$ is equal to a/b in the simplest fraction form. What is $b - a$?
- A. 13 B. 23 C. 26 D. 46 E. None of these.
19. A triangular board is placed on the xy -plane so that the vertices are located at $(2, 1)$, $(5, -3)$, and $(5, 2)$. Where is the center of the triangle (a.k.a. the centroid)?
- A. $(4, 0)$ B. $(4, 3/2)$ C. $(4, -1)$ D. $(0, 4)$ E. None of these.
20. If the sum of two numbers is 23, and their difference is 7, what is their product?
- A. 132 B. 130 C. 120 D. 60 E. None of these.
21. How many trailing zeroes are at the end of the number $25!$?
- A. 5 B. 6 C. 7 D. 8 E. None of these.
22. If r_1 and r_2 are the solutions to the quadratic equation $x^2 + 2x - 5 = 0$, then $(r_1 - r_2)^2 =$
- A. 4 B. 6 C. 12 D. 14 E. 24
23. The sum of $\frac{2015}{2016}$ and its reciprocal is
- A. < 1 B. < 2 C. $= 2$ D. > 2 E. > 3

24. If $\sin(x) = 1/5$ and $0^\circ < x < 90^\circ$, what is the value of $\sin(2x)$?
- A. $\frac{2\sqrt{6}}{25}$ B. $\frac{\sqrt{6}}{25}$ C. $\frac{24}{25}$ D. $\frac{4\sqrt{6}}{25}$ E. None of these.
25. If $\frac{a+2}{3} + \frac{2a-7}{3} = b$, what is the value of $a - b$?
- A. $-\frac{5}{3}$ B. $\frac{3}{5}$ C. $-\frac{3}{5}$ D. $\frac{5}{3}$ E. None of these.
26. James didn't have school on one Monday morning because of the snow storm over the weekend, so he got bored and started to factor numbers for fun. He noticed that $2015 = 5 \times 13 \times 31$, and $2016 = 2^5 \times 3^2 \times 7$. He then counted the number of positive factors for both 2015 and 2016, and noticed that 2015 has x such factors and 2016 has y . What is $y - x$?
- A. 36 B. 5 C. 8 D. 28 E. None of these.
27. $1 + 3 + 5 + \dots + 2015 =$
- A. 2015^2 B. 2015×2016 C. 2015×1008 D. 1007^2 E. 1008^2
28. $2015^2 - 2014^2 =$
- A. 8116421 B. 4060225 C. 4096 D. 4029 E. None of these.
29. The base 8 number 2015_8 is equal to (in base 10)
- A. 1101 B. 525 C. 585 D. 1037 E. None of these.
30. Jonathan lives five blocks south and five blocks west of his school. Each day he goes to school by walking east and north only because he wants to walk the shortest distance. Assume all the streets are safe for walking and he can only make turns at intersections. How many different routes are possible from his house to the school?
- A. 504 B. 252 C. 126 D. 70 E. None of these.
31. If $\log_3 x = \sqrt{2}$, what is the value of $\log_9 \sqrt{x}$?
- A. $\frac{\sqrt{2}}{2}$ B. $\frac{\sqrt{2}}{4}$ C. $\frac{\sqrt{\sqrt{2}}}{2}$ D. $\frac{\sqrt{\sqrt{2}}}{4}$ E. None of these.

32. A square $ABCD$ has side length $AB = 16\text{cm}$. Two semicircles with diameters on the sides AB and BC intersect inside the square. What is the area of the overlapping region between the two semicircles?
- A. $32\pi - 64$ B. $16\pi - 32$ C. $32\pi - 16$ D. $64\pi - 32$ E. None of these.
33. Dr. Kent has a favorite mug, which is of a rather strange shape that's hard to describe (he has "unique" artistic taste compared to most people). He likes it so much that he wanted to get another one that looks exactly the same, except twice as tall, and proportional in all other directions (in math terms: he wants the new mug to be similar to the original one). If the original mug holds 12 fl oz of water, how much water can the new mug hold?
- A. 24 fl oz B. 36 fl oz C. 48 fl oz D. 96 fl oz E. None of these.
34. How many different ways can n consecutive positive integers have a sum equal to 15? ($n > 1$)
- A. 1 B. 2 C. 3 D. 4 E. 5
35. Euler's formula says $e^{ix} = \cos x + i \sin x$, where $i = \sqrt{-1}$. What is $e^{i\pi}$ according to Euler's formula?
- A. 1 B. -1 C. 0 D. $-i$ E. i
36. You are bored in English class and so you start to fill in the cartesian plane as shown in the diagram.



- In which layer will the number 2015 appear? Here, we consider 1, 2, 3 and 4 the first layer, and 5, 6, 7, 8 \dots , 15, 16 the second layer, etc.
- A. 20th B. 21st C. 22nd D. 23rd E. 24th
37. How many real solutions does the equation $x^3 + 2015x + 1 = 0$ have?
- A. 0 B. 1 C. 2 D. 3 E. None of these.

- 38.** What is the minimum number of fair coin tosses needed to make the probability of at least two consecutive outcomes being the same (i.e., HH or TT shows up at least once in the process) greater than 99%? (H stands for heads, and T stands for tails for the coin toss outcome.)
- A. 6 B. 7 C. 8 D. 9 E. 10
- 39.** Dr. Johnson got to his office at 8:00AM one morning, and a little while later (before 9:00AM), with his extremely sharp angle recognition skills, he noticed the analog clock on the wall shows a 75° angle between the hour hand and the minute hand. What time was it when this happened?
- A. 8:20AM B. 8:25AM C. 8:30AM D. 8:35AM E. None of these.
- 40.** A palindrome is a positive integer that remains the same when all the digits are reversed. For example, 34243 is a palindrome. Some numbers (but not all) can be written as the sum of two palindromes. For example, $2015 = 1551 + 464$. What is the smallest integer greater than 2015 that CANNOT be written as the sum of two palindromes?
- A. 2016 B. 2017 C. 2018 D. 2019 E. 2020

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