2016 Marywood Mathematics Contest

Level II

Sponsored by

iMACS

The Marywood Math and Computer Science Club

Marywood University

and

Mu Alpha Theta

March 19, 2016

Directions:

1. This exam consists of 40 questions on 6 pages. Please check to make sure that you have all the pages.

2. No calculator or any other electronic device is allowed on this exam.

3. Allot your time accordingly. This is a 60-minute test. Do not spend too much time on any one problem. If a question seems to be too difficult, make your best possible guess. Your score will be the number of correct responses.

4. On the scantron form provided for you, darken in the space corresponding to the correct answer. Please mark all answers carefully and erase completely when changing an answer. Mark **only one answer** for each question. Only those answers on the answer sheet will be counted.

5. There is a sheet of blank paper on the last page which you can tear off and use as scratch paper. You may also use the back of the pages.

6. NOTE: **In order to ensure uniformity, proctors are NOT allowed to answer any questions pertaining to specific problem content.**

7. May the force be with you.

Please do NOT open the test until you are told to do so.

1. If x is an odd integer, which of the following is even?

A. $2x+1$ **B.** $2x+3$ **C.** $x+x-1$ **D.** $2(x+1)$ **E.** None of these

2. Find t so that the three points are collinear: $(-2,5)$, $(0,t)$, $(1,1)$

A. $\frac{4}{3}$ **B.** $\frac{5}{3}$ **C.** $\frac{7}{3}$ **D.** $\frac{8}{3}$ **E.** None of these

3. Find the distance between the points $(5,4)$ and $(2,8)$.

A. 5 **B.** **C.** **D.** **E.** None of these

4. If $a\triangle b = a^b + b^a$, then $(2\triangle 3)\triangle 1 =$

A. 12 **B.** 18 **C.** 22 **D.** 48 **E.** None of these

5. An explorer is searching for buried treasure. To reach the region where the treasure is buried, she must pass through three gates, leaving half her money at each gate. With how much money must she begin in order to have \$144 left to buy supplies when she reaches the treasure?

A. \$288 **B.** \$1,050 **C.** \$1,152 **D.** \$2,304 **E.** None of these

6. If $\sin\theta = \frac{\sqrt{3}}{2}$, then $\theta =$

A. $\frac{\pi}{2}$ **B.** $\frac{\pi}{3}$ **C.** $\frac{\pi}{4}$ **D.** $\frac{\pi}{6}$ **E.** None of these

7. How many unique real solutions does the equation $x^3 - 4x = 0$ have?

A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** None of these

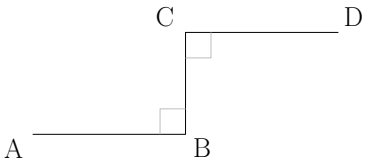
8. You toss a fair coin 5 times. What is the probability that the you do not get 5 tails.

A. $\frac{1}{32}$ **B.** $\frac{1}{5}$ **C.** $\frac{31}{32}$ **D.** $\frac{1}{10}$ **E.** None of these

9. A room has six doorways. In how many ways can a person enter the room and then leave the room without using the same door?

A. 5 **B.** 6 **C.** 25 **D.** 36 **E.** None of these

10. If AB is perpendicular to BC, BC is perpendicular to CD, $AB = 8$, $BC = 5$, and $CD = 4$, then what is the shortest path from A to D?



A. 12 **B.** 13 **C.** 14 **D.** 15 **E.** None of these

11. If -3 is a solution to $x^2 + hx + 27 = 0$, then $h =$

A. 12 **B.** 13 **C.** 14 **D.** 15 **E.** None of these

12. $\log_2\left(\frac{1}{8}\right) =$

A. $\frac{1}{3}$ **B.** $-\frac{1}{3}$ **C.** 3 **D.** -3 **E.** None of these

13. If $|x - 3| \leq 4$, then

A. $1 \leq x \leq 7$ **C.** $-1 < x < 7$ **E.** None of these
B. $-7 \leq x \leq -1$ **D.** $-7 < x < 1$

14. The domain of the function $f(x) = \frac{1}{\sqrt{3-x}}$ is

A. $x < 3$ **B.** $x \leq 3$ **C.** $x > 3$ **D.** $x \geq 3$ **E.** None of these

15. How many cubic centimeters (cc) of water must be added to 100 cc of an 80% solution of boric acid to reduce it to a 50% solution?

A. 30 **B.** 40 **C.** 50 **D.** 60 **E.** None of these

16. Express the following product as a reduced fraction

$$\left(1 - \frac{1}{2016}\right)\left(1 - \frac{1}{2015}\right)\left(1 - \frac{1}{2014}\right)\cdots\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{1}\right)$$

A. $\frac{2015}{2016}$ **B.** $\frac{1}{2016}$ **C.** $\frac{503}{1008}$ **D.** $\frac{251}{504}$ **E.** None of these

17. Find the x-coordinate of the solution of the system

$$\begin{aligned} 3x + 4y &= 3 \\ x - 2y &= 6 \end{aligned}$$

A. 1 **B.** 2 **C.** 3 **D.** 4 **E.** None of these

18. $\dfrac{1}{\tan\theta} + \dfrac{1}{\cot\theta} =$

A. $\dfrac{1}{\sin\theta\cos\theta}$ **B.** $\dfrac{\sin\theta}{\cos\theta}$ **C.** $\sin\theta\cos\theta$ **D.** $\dfrac{\cos\theta}{\sin\theta}$ **E.** None of these

19. Solve $x^2 - 4x + 29 = 0$

A. $-4 \pm 2i$ **B.** $4 \pm 2i$ **C.** $-2 \pm 4i$ **D.** $2 \pm 4i$ **E.** None of these

20. $(5 - 3\sqrt{5})(3 + \sqrt{5}) =$

A. $8\sqrt{5}$ **B.** $-4\sqrt{5}$ **C.** $3\sqrt{5}$ **D.** 15 **E.** None of these

21. For all non-zero real numbers x and y such that $x - y = xy$, $\dfrac{1}{x} - \dfrac{1}{y} =$

A. $\dfrac{1}{xy}$ **B.** $\dfrac{1}{x-y}$ **C.** 1 **D.** -1 **E.** None of these

22. If $x^2 + 2x + n > 10$ for all real numbers x, then which of the following conditions must be true?

A. $n > 11$ **B.** $n = 10$ **C.** $n = 11$ **D.** $n < 10$ **E.** None of these

23. 12 knights of Camelot are gathered around a large round table. Out of this group, 1/2 belong to the "Sir Lancelot Fan Club," 1/3 belong to the "Sir Galahad" fan club, and 1/4 belong to both clubs. How many belong to neither fan club?

A. 2 **B.** 3 **C.** 4 **D.** 5 **E.** None of these

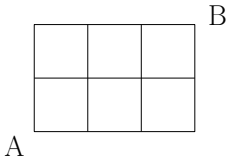
24. Marywood Field Hocky team won 30% of 60 consecutive games it played. Under the leadership of thier new captain, Susan, this team had a winning streak that raised its average to 50%. How many games did the team play in its winning streak to obtain this average?

A. 18 **B.** 24 **C.** 30 **D.** 36 **E.** None of these

25. If the mean of x, 3, $4x-3$, $x+4$, -16, 9, and $x-4$ is 4, what is x?

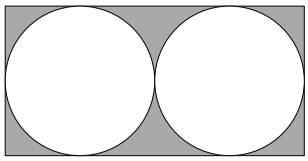
A. 6 **B.** 8 **C.** 5 **D.** 10 **E.** None of these

26. How many paths are there from A to B which only move right or up and follow the lines?



A. 5 **B.** 6 **C.** 7 **D.** 10 **E.** None of these

27. The two congruent circles below are tangent to the sides of the rectangle and tangent to each other. If the radius of each is equal to 2, what is the area of the shaded region?



A. $32-8\pi$ **B.** $8-4\pi$ **C.** $16\pi-8$ **D.** $16(\pi-1)$ **E.** None of these

28. The point $A=(2,3)$ is reflected about the x-axis to a point B. Then B is reflected about the line $y=x$ to a point C. What is the area of the triangle ABC?

A. 12 **B.** 18 **C.** 24 **D.** 30 **E.** None of these

29. This problem and the next deal with converting a number from one base to base 10. Recall that $523_{\text{base }10} = 5\cdot 10^2 + 2\cdot 10 + 3 = 523$, while $523_{\text{base }6} = 5\cdot 6^2 + 2\cdot 6 + 3 = 195$

$231_{\text{base }5} =$

A. 66 **B.** 91 **C.** 30 **D.** 231 **E.** None of these

30. $101_{\text{base }(11_{\text{base }2})} =$

A. 10 **B.** 5 **C.** 123 **D.** 42 **E.** None of these

31. What is the difference between the sum of all even integers from 1 to 100 and the sum of all odd integers from 1 to 100?

A. 1 **B.** 2 **C.** 0 **D.** 4 **E.** None of these

32. In a game of Gorf, three gligs and two glugs score 21 points, while two gligs and 5 glugs score 25 points. How many points do four gligs and three glugs score?

A. 28 **B.** 29 **C.** 30 **D.** 31 **E.** None of these

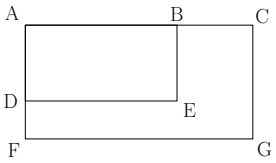
33. $(3+2i)^2 =$

A. $13-12i$ **B.** $13+12i$ **C.** $9-4i$ **D.** $9+4i$ **E.** None of these

34. What is the smallest positive integer k such that $2x(kx-4)-x^2+6=0$ has no real roots.

A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** None of these

35. $ACGF$ and $ABED$ are both rectangles with $AB=\frac{2}{3}AC$ and $AD=\frac{2}{3}AF$. What is the ratio of the area of $ACGF$ to the area of $ABED$?



A. $\frac{1}{9}$ **B.** $\frac{2}{3}$ **C.** $\frac{4}{9}$ **D.** $\frac{2}{3}$ **E.** None of these

36. If $\log(\log(\log(\log(x))))=0$, where $\log(10)=1$, then $x=$

A. 10 **B.** $10^{10,000}$ **C.** $10^{10,000,000}$ **D.** $10^{10,000,000,000}$ **E.** None of these

37. How many positive divisors does 6! have?

A. 4 **B.** 6 **C.** 10 **D.** 20 **E.** None of these

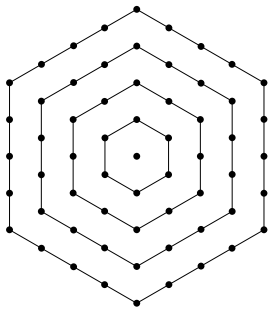
38. If b varies over all real numbers, upon which curve do the vertexes of the parabolas with equations $y=x^2+bx$ lie?

A. a line **B.** a circle **C.** a parabola **D.** a hyperbola **E.** None of these

39. For positive integers n, it is know that $1+3+5\cdots+(2n-1)=n^2$.
Evaluate $\dfrac{1+3+5+\cdots+739}{741+743+\cdots+1479}$

A. $\frac{1}{2}$ **B.** $\frac{1}{3}$ **C.** $\frac{1}{4}$ **D.** $\frac{1}{5}$ **E.** None of these

40. Consider the hexagonal array shown below. Then n^{th} hex number is defined as the total number of dots in the first n layers. The first seven hex numbers are 1, 7, 19, 37, 61, 91, and 127.



Find the 50^{th} hex number.

A. 294 **B.** 7,351 **C.** 7,650 **D.** 7,651 **E.** None of these

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